4/19/2001

Calculus Exam (§5.7∼7.1)

1. Find the solution to the initial value problem $e^{x^2}y' = xe^y$ with the initial condition $y(0) = 0$.

2. (a) Evaluate: $\sin[\arccos(-\frac{5}{\sqrt{74}})]$.

   (b) Complete: $\text{arccot}(\text{x} - 1) = \arctan$ _____.

3. Evaluate:

   (a) $\int \frac{dx}{\sqrt{8 + 2x - x^2}}$.

   (b) $\int \frac{2 - x}{\sqrt{4 - x^2}} dx$.

4. Differentiate: $f(x) = \coth^{-1}(\text{e}^{\text{x}})$.

5. Consider $f(x) = x^3 + x^2 + 1$ and $g(x) = -x^2 + 3x + 1$, compute the area of the region bounded by f and g.

6. (a) Find the volume of the solid formed by revolving the region bounded by the graph of $y = \frac{1}{x}, y = x^2$, and x=2 about the x-axis.

   (b) Consider the region bounded by the graphs of $y = \frac{1}{x}, y = x^2$, and x=2, calculate the volume of the solid formed when this region is revolved about the line $x = 2$.

7. Find the arc length of the graph of $f(x) = (\frac{2}{3})(x - 2)^{3/2}$ on the interval [2,4].

8. Find the work done in filling an upright cylindrical tank of radius 3 feet and height 10 feet with a liquid that weighs 40 pounds per cubic foot. Assume that the liquid is pumped into the tank through a hole in the bottom of the tank.

9. Consider the plane region bounded by the graphs $f(x) = -x^2 + 2$ and $g(x) = 1$, find the centroid of the region.

10. Evaluate the integral: $\int \frac{x}{(x + 2)^2} dx$.