6/19/2001

Calculus Exam (§7.2~8.9)

1. Evaluate the integral: $\displaystyle\int_0^{\sqrt{3}/2} \frac{t^2}{(1-t^2)^{3/2}}dt$.

2. Evaluate the limit using L'Hôpital's Rule: $\displaystyle\lim_{x\to\infty} \frac{(\ln x)^n}{x^m} \quad n, m > 0$.

3. Fibonacci sequence is defined recursively by $a_{n+2} = a_n + a_{n+1}$, where $a_1 = 1$ and $a_2 = 1$, The golden ratio ρ can be defined by $\lim\limits_{n\to\infty} b_n = \rho$, $b_n = \frac{a_{n+1}}{a_n}$, $b_n = 1 + \frac{1}{b_{n-1}}$. Show that $\rho = 1 + 1/\rho$ and solve this equation for ρ.

4. Determine the convergence or divergence of the series: $\sum\limits_{n=1}^{\infty} n^k e^{-n}$, k is a positive integer.

5. Determine the number of terms required to approximate the sum of the convergent series with an error of less than 0.001:
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}.$$

6. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$. (Be sure to include a check for convergence at the endpoints of the interval.)

7. Find the sum of the convergent series, $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\frac{2^n}{5^n n}$. Explain how you obtain the sum.

8. Find the limit analytically: $\lim\limits_{x\to 0^+} (\tan x)^x$.

9. Evaluate the improper integral: $\displaystyle\int_0^2 \frac{x}{3-2x}dx$.

10. Let f be the function $f(x) = e^{-x^2}$.
    (a) Use the power series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ to evaluate the integral: $\int e^{-x^2}dx$.
    (b) Use the result of part(a) to approximate the definite integral with an error of less than 0.0001: $\displaystyle\int_0^{0.5} e^{-x^2}dx$.

11. Evaluate: $\int \arcsin t dt$.

(a)$\frac{1}{\sqrt{1-t^2}}+C$ (b)$-\arccos t+C$ (c)$t\arcsin t+\sqrt{1-t^2}+C$

(d)$t\arcsin t-\ln\sqrt{1-t^2}+C$ (e) None of these

12. Evaluate: $\int_{\pi/8}^{\pi/4}\sin^2 2\theta d\theta$.

(a)$\frac{4-\sqrt{2}}{12}$ (b)$\frac{4-\sqrt{2}}{24}$ (c) $\frac{\pi-2}{16}$ (d)$\frac{\pi+2}{16}$ (e) None of these

13. Use the method of partial fractions to evaluate the indefinite integral: $\dfrac{3+10(\ln x)^2}{x(\ln x)^3+x\ln x}dx$.

(a)$3\ln\mid\ln x\mid+\frac{7}{2}\ln\mid x^2+1\mid+C$

(b)$3\ln\mid(\ln x)^3+\ln x\mid+C$

(c)$3\ln x+\frac{7}{2}\ln(x^2+1)+C$

(d)$3\ln\mid\ln x\mid+\frac{7}{2}\ln[(\ln x)^2+1]+C$

(e) None of these