Graduate School of Business, University of Chicago
Business 41202, Spring Quarter 2002, Mr. Ruey S. Tsay

Midterm

GSB Honor Code:
I pledge my honor that I have not violated the Honor Code during this examination.

Signiture: **Name**: **ID**:

Notes:

- Open notes and books.
- Write the answer in the blank space provided for each question.
- Manage your time carefully and answer as many questions as you can.
- The exam has 6 pages and the computer output has 7 pages.

Problem A: (40 pts) Answer briefly the following questions.

1. What is the simple return R_t in percentage if the log return, in percentage, is $r_t = 2.25$?

2. What is the log return r_t in percentage if the simple return, in percentage, is -1.25?

3. Is it true or false that the daily log returns of a stock tend to have positive excess kurtosis?

4. Is it true or false that the empirical distribution of daily log returns of a stock tends to have a higher peak than the corresponding normal distribution?

5. Suppose that the data $\{r_t | t = 1, \cdots, T\}$ are available. Define the sample skewness and sample excess kurtosis of the data.

6. Define the lag-1 sample autocorrelation function of r_t of Problem 5. This correlation measures the linear dependence of r_t on which variable?

7. Describe a test statistic, including the decision rule, that can be used to test the hypothesis $H_o : \rho_1 = \rho_2 = \rho_3 = \rho_4 = 0$ vs $H_a : \rho_i \neq 0$ for some $i \in \{1, \cdots, 4\}$, where $\rho_i =$ is the lag-i serial correlation of a log return r_t. Assume that the return series is weakly stationary.

8. Describe a method to test the null hypothesis $H_0 : \rho_4 = 0$ vs $H_a : \rho_4 \neq 0$, where ρ_4 is the lag-4 autocorrelation of a quarterly earning series x_t. What is the test statistic? What is the reference distribution?

9. Consider the sequence $\{-0.02, 0.01, 0.01, -0.01, 0.02\}$. (a) If the sequence denotes the daily log returns of a stock within a given week, what is the corresponding weekly log return of the stock? (b) If the sequence represents the daily simple returns of a stock within a given week, what is the corresponding weekly simple return of the stock?

10. Why are we concerned about a large excess kurtosis? Give a specific reason.

11. **For Problems 11 to 13**, assume that the monthly log return of an asset follows the model

$$r_t = 0.0065 + 0.086 r_{t-1} - 0.125 r_{t-3} + a_t, \quad \sigma_a = 0.058.$$

What is the expected value of the return?

12. Suppose that $r_{792} = 0.112$, $r_{791} = -0.044$ and $r_{790} = 0.012$. Compute the 1-step ahead forecast of r_t at the forecast origin $t = 792$.

13. What is the forecast error of the above 1-step ahead prediction? What is the 95% interval of the 1-step ahead forecast at the forecast origin $t = 792$?

14. Suppose the log price of a stock follows the model $p_t = 0.001 + p_{t-1} + a_t$, where $\{a_t\}$ is a white noise series. Is it true or false that long term forecasts of the log price will increase linearly with the forecast horizon at a given forecast origin?

15. **For questions 15 to 16**, assume that the log return r_t follows the MA(2) model

$$r_t = 0.257 + a_t - 0.08a_{t-2},$$

where $\{a_t\}$ is a white noise series with mean zero and variance 8.0. Is it true or false that the lag-1 autocorrelation of r_t is zero. Why?

16. What is the 3-step ahead forecast of r_t at the forecast origin $t = 100$?

17. State two financial applications in which seasonal models are useful.

18. State two weaknesses of an ARCH model in modeling the volatility of an asset return.

19. State two financial applications in which volatility of an asset return is useful.

20. Describe two reasons that may explain the existence of serial correlations in a daily asset return series.

Problem B. (20 pts) This problem is concerned with daily log returns of Wal-Mart stock from January 1991 to December 2001 for 2775 observations. The returns are in percentage and the necessary SCA and RATS outputs are attached. Answer the following questions:

1. Is the average daily log return significantly different from zero? Use a 5% test to answer the question.

2. What is the excess kurtosis of the data? Is it significantly different from zero at the 5% level?

3. ACF of the log returns shows some significant serial correlations. An AR(3) model is estimated for the log return series. Write down the fitted model, including the standard error of the residuals.

4. (4 pts) Do the daily log returns show any ARCH effects? Why? Provide the test statistic you used to draw the conclusion.

5. An AR(3)-GARCH(1,1) model is applied to the data. Write down the fitted mean and volatility equations.

6. (4 pts) Is the AR(3)-GARCH(1,1) model adequate? Why?

Problem C. (20 pts) The daily log returns of Wal-Mart stock continued. Some RATS output is attached. Answer the following questions:

1. A Gaussian AR(3)-GARCH(1,1)-M model is applied to the data. What are the fitted mean and volatility equations? [Use 3 decimal points for parameter estimates.]

2. (4 pts) Is the fitted model adequate? What are the test statistics used to check the mean and volatility equations? You may use the 5% significance level to draw the conclusion.

3. Is the risk premium parameter significant at the 5% level? Why?

4. (4 pts) Compute 1-step ahead mean and volatility forecasts of the model at the forecast origin $t = 2775$.

5. An AR(3)-EGARCH(1,1) model is also applied to the data, where $g(\epsilon_t)$ is defined as in the text, i.e. $g(\epsilon_t) = \theta\epsilon_t + \gamma(|\epsilon_t| - \sqrt{2/\pi})$. Based on the estimates, write down the $g(\epsilon_t)$ function. Are both estimates of θ and γ significant at the 5% level?

6. The volatility equation used is

$$\ln(\sigma_t^2) = (1 - \alpha_1)\alpha_0 + \alpha_1 \ln(\sigma_{t-1}^2) + g(\epsilon_{t-1}) + \beta_1 g(\epsilon_{t-2}).$$

   What is the fitted volatility equation if both ϵ_{t-1} and ϵ_{t-2} are negative?

Problem D. (20 pts) This problem is concerned with Monday effects on the daily log returns of HP stock. The same data as that of Homework Assignment #3 are used. However, the model is extended to include volatility equation. The model used is

$$
\begin{aligned}
r_t &= p_0 M_t + a_t \\
\sigma_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + m_1 M_t
\end{aligned}
$$

where M_t is the dummy variable for Mondays, i.e. $M_t = 1$ if t is Monday and it is zero otherwise. Answer the following questions. Use 5% significance level in all tests.

1. Write down the fitted model.

2. Is there a Monday effect on the mean equation? Why?

3. Is there a Monday effect on the volatility equation? Why?

4. What is the unconditional variance of a_t if t is not a Monday? What is the unconditional variance of a_t on Mondays?

5. The first trading day of Year 2000 was Monday January 3. Use the fitted model to compute 1-step ahead forecasts for the return and its volatility at the end of Year 1999 ($t = 5056$).