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Business 41202, Spring Quarter 2002, Mr. Ruey S. Tsay

Solutions to Midterm

Problem A: (40 pts) Answer briefly the following questions.

1. What is the simple return R_t in percentage if the log return, in percentage, is $r_t = 2.25$?

   Answer: $R_t = 100(e^{2.25/100} - 1) = 2.276$.

2. What is the log return r_t in percentage if the simple return, in percentage, is -1.25?

   Answer: $r_t = 100\ln(1 + \frac{-1.25}{100}) = -1.258$.

3. Is it true or false that the daily log returns of a stock tend to have positive excess kurtosis?

   Answer: True

4. Is it true or false that the empirical distribution of daily log returns of a stock tends to have a higher peak than the corresponding normal distribution?

   Answer: True

5. Suppose that the data $\{r_t | t = 1, \cdots, T\}$ are available. Define the sample skewness and sample excess kurtosis of the data.

   Answer: $S = \frac{\sum_{t=1}^T (r_t - \bar{r})^3}{[\sum_{t=1}^T (r_t - \bar{r})^2]^{3/2}}$ and $K^* = \frac{\sum_{t=1}^T (r_t - \bar{r})^4}{[\sum_{t=1}^T (r_t - \bar{r})^2]^2} - 3$, where $\bar{r} = \frac{1}{T}\sum_{t=1}^T r_t$. (Other equivalent definitions are fine.)

6. Define the lag-1 sample autocorrelation function of r_t of Problem 5. This correlation measures the linear dependence of r_t on which variable?

   Answer: $\hat{\rho}_1 = \frac{\sum_{t=2}^T (r_t - \bar{r})(r_{t-1} - \bar{r})}{\sum_{t=1}^T (r_t - \bar{r})^2}$. This correlation measures the linear dependence of r_t on r_{t-1}.

7. Describe a test statistic, including the decision rule, that can be used to test the hypothesis $H_o: \rho_1 = \rho_2 = \rho_3 = \rho_4 = 0$ vs $H_a: \rho_i \neq 0$ for some $i \in \{1, \cdots, 4\}$, where $\rho_i =$ is the lag-i serial correlation of a log return r_t. Assume that the return series is weakly stationary.

   Answer: Use $Q(4) = T(T+2)\sum_{i=1}^4 \frac{\hat{\rho}_i^2}{T-i}$. Compare it with a χ_4^2 distribution. Reject H_o if $Q(4) > \chi_4^2(0.95)$, where $\chi_4^2(0.95)$ denotes the 95th percentile of a chi-squared distribution with 4 degrees of freedom.

8. Describe a method to test the null hypothesis $H_0: \rho_4 = 0$ vs $H_a: \rho_4 \neq 0$, where ρ_4 is the lag-4 autocorrelation of a quarterly earning series x_t. What is the test statistic? What is the reference distribution?

   Answer: Use t ratio. $t = \frac{\hat{\rho}_4}{\text{std}(\hat{\rho}_4)} = \sqrt{T}\hat{\rho}_4$. The reference distribution is $N(0,1)$.

9. Consider the sequence $\{-0.02, 0.01, 0.01, -0.01, 0.02\}$. (a) If the sequence denotes the daily log returns of a stock within a given week, what is the corresponding weekly log return of the stock? (b) If the sequence represents the daily simple returns of a stock within a given week, what is the corresponding weekly simple return of the stock?

   Answer: Weekly log return is the sum, which is 0.01. The corresponding simply return is $R = e^{0.01} - 1 = 0.01$.

10. Why are we concerned about a large excess kurtosis? Give a specific reason.

    Answer: Existence of outliers or efficiency in estimation.

11. **For Problems 11 to 13**, assume that the monthly log return of an asset follows the model

    $$r_t = 0.0065 + 0.086 r_{t-1} - 0.125 r_{t-3} + a_t, \quad \sigma_a = 0.058.$$

    What is the expected value of the return?

    Answer: $E(r_t) = \frac{0.0065}{1-0.086+0.125} = 0.00626$.

12. Suppose that $r_{792} = 0.112$, $r_{791} = -0.044$ and $r_{790} = 0.012$. Compute the 1-step ahead forecast of r_t at the forecast origin $t = 792$.

    Answer: $r_{792}(1) = 0.0065_0.086(0.112) - 0.125(0.012) = 0.01463$.

13. What is the forecast error of the above 1-step ahead prediction? What is the 95% interval of the 1-step ahead forecast at the forecast origin $t = 792$?

    Answer: Error $= a_{793}$. 95% interval forecast is $0.01463 \pm 1.96(0.058)$, i.e. $(-0.0991, 0.1283)$. If you use 2 as the critical value, the interval is $(-0.1014, 0.1306)$.

14. Suppose the log price of a stock follows the model $p_t = 0.001 + p_{t-1} + a_t$, where $\{a_t\}$ is a white noise series. Is it true or false that long term forecasts of the log price will increase linearly with the forecast horizon at a given forecast origin?

    Answer: True, because the time slope 0.001 is positive.

15. **For questions 15 to 16**, assume that the log return r_t follows the MA(2) model

    $$r_t = 0.257 + a_t - 0.08 a_{t-2},$$

    where $\{a_t\}$ is a white noise series with mean zero and variance 8.0. Is it true or false that the lag-1 autocorrelation of r_t is zero. Why?

    Answer: True, because $t_{t-1} = 0.257 + a_{t-1} - 0.08 a_{t-3}$ shares no common shocks with r_t.

16. What is the 3-step ahead forecast of r_t at the forecast origin $t = 100$?

    Answer: $r_{100}(3) = 0.257$, the constant term.

17. State two financial applications in which seasonal models are useful.

    Answer: Any two of the the following: Pricing energy related derivative, forecasting quarterly earning, or analysis of intraday financial data.

18. State two weaknesses of an ARCH model in modeling the volatility of an asset return.

    Answer: Any two of the following: symmetric between positive and negative shocks, restrictive, providing no explanation, and tendence to over-predict volatility after an isolated big shock.

19. State two financial applications in which volatility of an asset return is useful.

    Answer: Any two of the following: pricing derivatives, asset allocation, interval forecast, and risk management (Value at Risk)

20. Describe two reasons that may explain the existence of serial correlations in a daily asset return series.

    Answer: Any two of the following: non-synchronous trading, bid-ask bounce, volatility induced risk premium.

Problem B. (20 pts) This problem is concerned with daily log returns of Wal-Mart stock from January 1991 to December 2001 for 2775 observations. The returns are in percentage and the necessary SCA and RATS outputs are attached. Answer the following questions:

1. Is the average daily log return significantly different from zero? Use a 5% test to answer the question.

   Answer: No, the t-ratio of the sample mean is 1.94, which is slightly smaller than 2.0. (One may say that it is marginally singinificant.)

2. What is the excess kurtosis of the data? Is it significantly different from zero at the 5% level?

   Answer: Excess kurtosis = 2.085 with t-ratio = $\frac{2.085}{0.0929}$ 22.45, which is highly significant.

3. ACF of the log returns shows some significant serial correlations. An AR(3) model is estimated for the log return series. Write down the fitted model, including the standard error of the residuals.

   Answer: $r_t = 0.0852 - 0.061r_{t-2} - 0.058r_{t-3} + a_t$ with $\sigma_a = 2.038$.

4. (4 pts) Do the daily log returns show any ARCH effects? Why? Provide the test statistic you used to draw the conclusion.

   Answer: Yes, because all Ljung-Box statistics for the squared residuals are highly significant, e.g. $Q(5) = 167$, which compared with χ_2^5 gives a p value of 0.

5. An AR(3)-GARCH(1,1) model is applied to the data. Write down the fitted mean and volatility equations.

   Answer: mean equation $r_t = 0.103 - 0.056r_{t-2} - 0.044r_{t-3} + a_t$, where $a_t = \sigma_t\epsilon_t$. The volatility equation is $\sigma_t^2 = 0.0307 + 0.0449a_{t-1}^2 + 0.9481\sigma_{t-1}^2$.

6. (4 pts) Is the AR(3)-GARCH(1,1) model adequate? Why?

   Answer: Yes, the Q-statistics of the standardized residuals [e.g. $Q(10) = 12.77(0.24)$] and squared standardized residuals [e.g. $Q(10) = 9.38(0.50)$] show no sign of model inadequency. The model also handles the skewness well, but some excess kurtosis remains.

Problem C. (20 pts) The daily log returns of Wal-Mart stock continued. Some RATS output is attached. Answer the following questions:

1. A Gaussian AR(3)-GARCH(1,1)-M model is applied to the data. What are the fitted mean and volatility equations? [Use 3 decimal points for parameter estimates.]

   Answer: The mean equation is $r_t = -0.056r_{t-2} - 0.043r_{t-3} + 0.028\sigma^2_{t-1} + a_t$. [You may state σ^2_t.] The volatility equation is $\sigma^2_t = 0.033 + 0.046a^2_{t-1} + 0.947\sigma^2_{t-2}$.

2. (4 pts) Is the fitted model adequate? What are the test statistics used to check the mean and volatility equations? You may use the 5% significance level to draw the conclusion.

   Answer: Yes, p-values of the $Q(10)$ and $Q(20)$ statistics for the standardized residuals and squared standardized residuals are all greater than 0.05.

3. Is the risk premium parameter significant at the 5% level? Why?

   Answer: Yes, its t-ratio is 3.12 with p-value 0.0018.

4. (4 pts) Compute 1-step ahead mean and volatility forecasts of the model at the forecast origin $t = 2775$.

   Answer: $r_{2775}(1) = -0.056(-0.034) - 0.043(0.378) + 0.028(2.469) = 0.055$. [If you use σ^2_{2775}, the answer is 0.052.] $\sigma^2_{2775}(1) = 0.033 + 0.046(-1.352)^2 + 0.947(2.370) = 2.361$.

5. An AR(3)-EGARCH(1,1) model is also applied to the data, where $g(\epsilon_t)$ is defined as in the text, i.e. $g(\epsilon_t) = \theta\epsilon_t + \gamma(|\epsilon_t| - \sqrt{2/\pi})$. Based on the estimates, write down the $g(\epsilon_t)$ function. Are both estimates of θ and γ significant at the 5% level?

   Answer: $g(\epsilon_t) = -0.066\epsilon_t + 0.161(|\epsilon_t| - 0.7979) = \begin{cases} -0.128 - 0.227\epsilon_t & \text{if } \epsilon_t < 0 \\ -0.128 + 0.095\epsilon_t & \text{otherwise.} \end{cases}$ Yes, both θ and γ estimates are significant at the 5% level.

6. The volatility equation used is

$$\ln(\sigma^2_t) = (1 - \alpha_1)\alpha_0 + \alpha_1 \ln(\sigma^2_{t-1}) + g(\epsilon_{t-1}) + \beta_1 g(\epsilon_{t-2}).$$

   What is the fitted volatility equation if both ϵ_{t-1} and ϵ_{t-2} are negative?

   Answer: $\ln(\sigma^2_t) = (1-\alpha_1)\alpha_0 + \alpha_1 \ln(\sigma^2_{t-1}) + [-0.128 - 0.227\epsilon_{t-1}] + \beta_1[-0.128 - 0.227\epsilon_{t-2}]$. Thus, $\sigma^2_t = (1 - 0.989)(1.519) + 0.989\ln(\sigma^2_{t-1}) - 0.128 - 0.227\epsilon_{t-1} - 0.034(-0.128 - 0.227\epsilon_{t-2}) = -0.068 + 0.989\ln(\sigma^2_{t-1}) - 0.227\epsilon_{t-1} + 0.077\epsilon_{t-2}$.

Problem D. (20 pts) This problem is concerned with Monday effects on the daily log returns of HP stock. The same data as that of Homework Assignment #3 are used. However, the model is extended to include volatility equation. The model used is

$$\begin{aligned} r_t &= p_0 M_t + a_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + m_1 M_t \end{aligned}$$

where M_t is the dummy variable for Mondays, i.e. $M_t = 1$ if t is Monday and it is zero otherwise. Answer the following questions. Use 5% significance level in all tests.

1. Write down the fitted model.

   Answer: The mean equation is $r_t = 0.249M_t + a_t$ and the volatility equation is $\sigma_t^2 = 0.553 + 0.082a_{t-1}^2 + 0.835\sigma_{t-1}^2 - 0.537M_t$.

2. Is there a Monday effect on the mean equation? Why?

   Answer: Yes, p value of the t-ratio is 0.00019.

3. Is there a Monday effect on the volatility equation? Why?

   Answer: Yes, p value of the t-ratio is 0.00194.

4. What is the unconditional variance of a_t if t is not a Monday? What is the unconditional variance of a_t on Mondays?

   Answer: Monday $E(a_t^2) = \frac{0.553-0.537}{1-0.082-0.835} = 0.193$. For other days, $E(a_t^2) = \frac{0.553}{1-0.082-0.835} = 6.663$.

5. January 3, 2000 was a Monday. Use the fitted model to compute 1-step ahead forecasts for the return and its volatility at the end of Year 1999.

   Answer: $r_{5056}(1) = 0.249$ and $\sigma_{5056}^2(1) = (0.553 - 0.537) + 0.082(-1.311)^2 + 0.835(7.452)$ $= 6.379$.