Statistics (II) Exam I, April 25, 2002

(1) Suppose that $X_1, X_2, \cdots, X_n$ are i.i.d. random variables on the interval [0,1] with the density function

$$f(x|\alpha) = \frac{\Gamma(2\alpha)}{\Gamma(\alpha)^2}[x(1-x)]^{\alpha-1}$$

where $\alpha > 0$. What is the asymptotic variance of the maximum likelihood estimate?

(2) Let $X_1, \cdots, X_n$ be an i.i.d. sample from a Rayleigh distribution with parameter $\vartheta > 0$:

$$f(x|\vartheta) = \frac{x}{\vartheta^2} e^{-x^2/2\vartheta^2}, \quad x \geq 0.$$

Find the method of moments estimate of ϑ.

(3) Let $X_1, \cdots, X_n$ be i.i.d. uniform on $[0, \vartheta]$. Compare the mean squared errors of the mle to that of the method of moment estimate.

(4) Let $X_1, \cdots, X_n$ be an i.i.d. sample from exponential distribution with the density function $f(x|\tau) = \frac{1}{\tau} e^{-x/\tau}, \quad 0 \leq x < \infty$. Find the forms of an approximate and an exact confidence intervals for τ.

(5) Find sufficient statistics for the gamma distribution.

(6) Consider testing goodness of fit for a multinomial distribution with two cells. Denote the number of observations in each cell by X_1 and X_2 and let the hypothesized probabilities be p_1 and p_2. Show that the Pearson's chi-square statistic is equal to $\frac{(X_1 - np_1)^2}{np_1(1-p_1)}$. What is the distribution of this statistic?

(7) Let $X_1, \cdots, X_n$ be a random sample from an exponential distribution with the density function $f(x|\vartheta) = \vartheta exp[-\vartheta x]$. Derive a likelihood ratio test of $H_0 : \vartheta = \vartheta_0$ versus $H_A : \vartheta \neq \vartheta_0$. Is it a uniformly most powerful test? Explain.

(8) Suppose that a test statistic T has a standard normal null distribution. If the test rejects for large values of $|T|$, what is the p-value corresponding to $T = 1.50$?