Time Series Analysis Exam II, May 27, 2002

(1) Find the autocovariance generating function of the following process

$$(1 - 1.4B + 0.6B^2)Z_t = (1 - 0.8B)a_t.$$

(2) Express the following process in the AR representation by actually finding the π weights,

$$(1 - B)^2 Z_t = a_t - 0.81a_{t-1} + 0.38a_{t-2}$$

(3) The statistic r_n is asymptotically distributed as $N(\rho, (1 - \rho^2)^2/n)$. Find a variance-stabilizing transformation of r_n, and find the approximate variance of the transformed variable.

(4) For the following two models,

(a) $(1 - \phi_1 B)(1 - B)Z_t = (1 - \vartheta_1 B)a_t$;

(b) $(1 - \phi_1 B - \phi_2 B^2)(Z_t - \mu) = a_t$,

find the l-step ahead forecasts $\hat{Z}_n(l)$ of Z_{n+l} and the variances of the two-step ahead forecast error.

(5) Show that the covariance between forecast errors from different origins is given by

$$Cov[e_n(l), e_{n-j}(l)] = \sigma_a^2 \sum_{i=j}^{l-1} \psi_i \psi_{i-j}, \quad l > j.$$

(6) Suppose that $Y_t = X_t + b_t$, where $(1 - B)X_t = (1 - \vartheta B)a_t,\ a_t \sim WN(0, \sigma_a^2)$ and $b_t = \phi b_{t-1} + \alpha_t,\ \alpha_t \sim WN(0, \sigma_\alpha^2)$, $\alpha_t's$ and $a_t's$ are independent. Show that Y_t has an ARIMA(p,d,q) covariance structure, and determine the values of p, d and q.

(7) Find the relation between the nonzero constant ϑ_0 in the model, $(1 - B)^d Z_t = \vartheta_0 + a_t$ and the coefficient of t^d in the deterministic trend $\alpha_0 + \alpha_1 t + \cdots + \alpha_d t^d$.