Time Series Analysis Exam I, Nov. 14, 2000

(1) Prove the process $Z_t = A\sin(2\pi\omega t+\theta)$ is covariance stationary, where A and ω are constants, and θ is a random variable which is uniformly distributed on $[0, 2\pi]$.

(2) Find a closed form solution of the difference equations : $Z_t - Z_{t-1} + Z_{t-2} - Z_{t-3} = 0$.

(3) Find the autocovariance function $\gamma_k, k = 0, 1, 2, \cdots$ of the model : $Z_t - 0.5Z_{t-1} = a_t$.

(4) Find the range of α such that the AR(2) process $Z_t = Z_{t-1} + \alpha Z_{t-2} + a_t$ is stationary.

(5) Let $\{X_t\}$ be the AR(2) process, $X_t = 0.7X_{t-1} - 0.1X_{t-2} + Z_t$, where $Z_t \sim WN(0, \sigma^2)$. Find the autocovariance and partial autocovariance functions of $\{X_t\}$.

(6) Consider the following AR(2) models, $Z_t - 0.8Z_{t-1} + 0.5Z_{t-2} = a_t$. Find the autocovariance function $\gamma_k, k = 0, 1, 2, \cdots$ of the model by assuming $\sigma_a^2 = 1$.

(7) Let $\{Y_t\}$ be a stationary time series with mean 0 and covariance function $\gamma_Y(k)$. Define $X_t = \sum_{j=0}^{\infty} \psi_j Y_{t-j}$, assuming $\sum_{j=0}^{\infty} |\psi_j| < \infty$. Show that X_t is also stationary with the following autocovariance function $\gamma_X(k) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \psi_j \psi_k \gamma_Y(h - j + k)$.

(8) Use the result of (7) to find the autocovariance function of the $ARMA(1, 1)$ process, $X_t - \phi X_{t-1} = Z_t + \theta Z_{t-1}$, where $|\phi| < 1$ and $Z_t \sim WN(0, \sigma^2)$.

(9) State all you know about (including the defintion, meaning $\cdots$) the partial autocorrelation function of a time series process.