Assignment #7

1.

Let $s_1(x) = 1 + c(x+1)^3$, $-1 \le x \le 0$, where c is a (real) parameter. Determine $s_2(x)$ on $0 \le x \le 1$ so that

$$s(x) := \begin{cases} s_1(x) & \text{if} \quad -1 \le x \le 0, \\ s_2(x) & \text{if} \quad 0 \le x \le 1 \end{cases}$$

is a natural cubic spline on [1,1] with knots at -1, 0, 1. How must c be chosen if one wants $s(1) = -1$?

2.

(a) Write a subroutine `tri(n,a,b,c,u,v)` for solving a tridiagonal (nonsymmetric) system

$$\begin{bmatrix} a_1 & c_1 & & & 0 \\ b_1 & a_2 & c_2 & & \\ & b_2 & a_3 & c_3 & \\ & & \ddots & \ddots & \ddots \\ & & & & c_{n-1} \\ 0 & & & b_{n-1} & a_n \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{n-1} \\ u_n \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{n-1} \\ v_n \end{bmatrix}$$

by Gauss elimination without pivoting. Keep the program short.

(b) Write a program for computing the natural spline interpolant $s_{\text{nat}}(f;\cdot)$ on an arbitrary partition $a = x_1 < x_2 < x_3 < \cdots < x_{n-1} < x_n = b$ of $[a,b]$.

Print $\{i, \text{errmax}(i);\ i = 1, 2, \ldots, n-1\}$, where

$$\text{errmax}(i) = \max_{1 \le j \le 51} \left| s_{\text{nat}}(f; x_{i,j}) - f(x_{i,j}) \right|, \quad x_{i,j} = x_i + \frac{j-1}{50}\,\Delta x_i.$$

(You will need the subroutine `tri`.) Test the program for cases in which the error is zero (what are these, and why?).

(c) Write a second program for computing the complete cubic spline interpolant $s_{\text{compl}}(f;\cdot)$ by modifying the program in (b) with a minimum of changes. Highlight the changes in the program listing. Apply (and justify) a test similar to that of (b).

(d) Run the programs in (b) and (c) for $[a,b] = [0,1]$, $n = 11$, and

(i) $x_i = \frac{i-1}{n-1}$, $i = 1, 2, \ldots, n$; $f(x) = e^{-x}$ and $f(x) = x^{5/2}$.

(ii) $x_i = \left(\frac{i-1}{n-1}\right)^2$, $i = 1, 2, \ldots, n$; $f(x) = x^{5/2}$.

Comment on the results.