Numerical Analysis **Assignment #1**

1. The following fortran program (attributed to CLEVE MOLER) estimates eps:

```
a=4./3.
b=a-1.
c=b+b+b
eps=abs(c-1.).
```

   Run the program with the corresponding double-precision statements appended to it and print the single- and double-precision eps.

2. (a) What is the distance $d(x)$ of a positive normalized floating-point number $x \in \mathbb{R}(t,s)$ to its next larger floating-point number:

$$d(x) = \min_{\substack{y \in \mathbb{R}(t,s) \\ y > x}} (y - x)\,?$$

   (b) Determine the relative distance $r(x) = d(x)/x$, with x as in (a), and give upper and lower bounds for it.

   (c) Prove that

$$max_{x \in \mathbb{R}(t,s)} |x| = (1 - 2^{-t}) 2^{2^s - 1} \qquad min_{x \in \mathbb{R}(t,s)} |x| = 2^{-2^s}$$

3. Assume that x_1^*, x_2^* are approximations to x_1, x_2 with relative errors E_1 and E_2, respectively, and that $|E_i| \leq E$, $i = 1, 2$. Assume further that $x_1 \neq x_2$.

   (a) How small must E (in dependence of x_1 and x_2) be in order to ensure that $x_1^* \neq x_2^*$?

   (b) Taking $\dfrac{1}{x_1^* - x_2^*}$ to approximate $\dfrac{1}{x_1 - x_2}$, obtain a bound on the relative error committed, assuming (i) exact arithmetic; (ii) machine arithmetic with machine precision eps. (Neglect higher-order terms in E_1, E_2, eps.)

4. Determine analytically the first value of the integer n for which x^n, $x = 10, 20, \ldots, 100$, overflows in single and in double precision. {*Hint*: Assume X_+ the largest machine-representable number, and identify this number for IEEE single precision ($t = 23$, $s = 8$) and IEEE double precision ($t = 52$, $s = 11$).}

5. Suppose, for $|x|$ small, one has an accurate value of $y = e^x - 1$ (obtained, e.g., by Taylor expansion). Use this value to compute accurately $\sinh x = \frac{1}{2}(e^x - e^{-x})$ for small $|x|$.