Numerical Analysis **Assignment #2**

1. The nth power of some positive (machine) number x can be computed

   (i) either by repeated multiplication by x, or

   (ii) as $x^n = e^{n \ln x}$.

   In each case, derive bounds for the relative error due to machine arithmetic, neglecting higher powers of the machine precision against the first power. Based on these bounds, state a criterion (involving x and n) for (i) to be better than (ii).

2. Indicate how you would accurately compute

$$(x+y)^{1/4} - y^{1/4}, \qquad x > 0, \qquad y > 0.$$

3. Compute the condition number of the following functions, and discuss any possible ill-conditioning.

   (a) $f(x) = x^{1/n} \quad (x > 0,\ n > 0 \text{ an integer})$

   (b) $f(x) = x - \sqrt{x^2 - 1} \quad (x > 1)$

   (c) $f(x_1, x_2) = \sqrt{x_1^2 + x_2^2}$

   (d) $f(x_1, x_2) = x_1 + x_2$

4. Show that $(\operatorname{cond} f \cdot g)(x) \leq (\operatorname{cond} f)(x) + (\operatorname{cond} g)(x)$.

5. Sum the series

$$\text{(a)} \quad \sum_{n=0}^{\infty} (-1)^n / n!^2 \qquad \text{(b)} \quad \sum_{n=0}^{\infty} 1/n!^2$$

   until there is no more change in the partial sums to within the machine precision. Generate the terms recursively. Print the number of terms required and the value of the sum. (Answers in terms of Bessel functions: (a) $J_0(2)$; (b) $I_0(2)$.)