Numerical Analysis **Assignment #3** **Due: October 20, 1999**

1. Given the natural number n, let $\xi = \xi(a)$ be the unique positive root of the equation $x^n = ae^{-x}$ $(a > 0)$. Determine the condition of ξ as a function of the parameter a; simplify the answer as much as possible. In particular, show that $(\operatorname{cond} \xi)(a) < 1/n$.

2. Let $f(x_1, x_2) = x_1 + x_2$ and consider the algorithm A given as follows,

$$f_A : \mathbb{R}^2(t,s) \to \mathbb{R}(t,s) \qquad y_A = \mathrm{fl}(x_1 + x_2).$$

   Estimate $\gamma(x_1, x_2) = (\operatorname{cond} A)(x)$, using any of the norms

$$\|x\|_1 = |x_1| + |x_2|, \quad \|x\|_2 = \sqrt{x_1^2 + x_2^2}, \quad \|x\|_\infty = \max\ (|x_1|, |x_2|).$$

   Discuss the answer in the light of the conditioning of f.

3. Euler's constant $\gamma = .57721566490153286\ldots$ is defined as the limit

$$\gamma = \lim_{n\to\infty} \gamma_n, \quad \text{where} \quad \gamma_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n.$$

   Assuming that $\gamma - \gamma_n \sim cn^{-d}$, $n \to \infty$, for some constants c and $d > 0$, try to determine c and d experimentally on the computer.

4. (P.J. Davis, 1993) Consider the series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{3/2} + k^{1/2}}$. Try to compute the sum to three correct decimal digits.

5. We know from calculus that

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e.$$

   What is the "machine limit"? Explain.