Numerical Analysis **Assignment #4** **Due: October 27, 1999**

1. Consider the data

$$f(t_i) = 1, \quad i = 1, 2, \ldots, N-1; \qquad f(t_N) = y \gg 1.$$

   (a) Determine the discrete L_∞ approximant to f by means of a constant c (polynomial of degree zero).

   (b) Do the same for discrete (equally weighted) least square approximation.

   (c) Compare and discuss the results, especially as $N \to \infty$.

2. Approximate the circular quarter arc γ given by the equation $y(t) = \sqrt{1-t^2}$, $0 \le t \le 1$ by a straight line ℓ in the least squares sense, using either the weight function $w(t) = (1-t^2)^{-1/2}$, $0 \le t \le 1$, or $w(t) = 1$, $0 \le t \le 1$. Where does ℓ intersect the coordinate axes in these two cases?
   {*Points of information*: $\int_0^{\pi/2} \cos^2 \theta d\theta = \frac{\pi}{4}$, $\int_0^{\pi/2} \cos^3 \theta d\theta = \frac{2}{3}$.}

3. Suppose you want to approximate $f(t)$ on $[a, b]$ by a function of the form $r(t) = \pi(t)/q(t)$ in the least squares sense with weight function w, where $\pi \in \mathbb{P}_n$ and q is a *given* function (e.g., a polynomial) such that $q(t) > 0$ on $[a, b]$. Formulate this problem as an ordinary polynomial least squares problem for an appropriate new function $\overline{f}$ and new weight function $\overline{w}$.

4. Given the recursion relation $\pi_{k+1}(t) = (t - \alpha_k)\pi_k(t) - \beta_k \pi_{k-1}(t)$, $k = 0, 1, 2, \ldots$, for the (monic) orthogonal polynomials $\{\pi_j(\,\cdot\,; d\lambda)\}$, and defining $\beta_0 = \int_{\mathbb{R}} d\lambda(t)$, show that $\|\pi_k\|^2 = \beta_0 \beta_1 \cdots \beta_k$, $k = 0, 1, 2, \ldots$.

5. Prepare plots of the Lebesgue function for interpolation, $\lambda_n(x)$, $-1 \le x \le 1$, for $n = 5, 10, 20$, with the interpolation nodes x_i being given by

   (a) $x_i = -1 + \frac{2i}{n}$, $\quad i = 0, 1, 2, \ldots, n$;

   (b) $x_i = \cos \frac{2i+1}{2n+2}\pi$, $\quad i = 0, 1, 2, \ldots, n$.

   Compute $\lambda_n(x)$ on a grid obtained by dividing each interval $[x_{i-1}, x_i]$, $i = 1, 2, \ldots, n$, into 20 equal subintervals. Plot $\log_{10} \lambda_n(x)$ in case (a), and $\lambda_n(x)$ in case (b). Comment on the results.