Numerical Analysis   Assignment #10

1. Suppose in the approximate formula

$$\int_0^1 \sqrt{x}\, f(x)dx \approx -\frac{2}{15} f(0) + \frac{4}{5} \int_0^1 f(x)dx$$

derived in class, the function f is known to be only once continuously differentiable, i.e., $f \in C^1[0,1]$.

   (a) Derive the appropriate Peano representation of the error functional Ef.

   (b) Obtain an estimate of the form $|Ef| \le c_0 \|f'\|_\infty$.

2. Determine the quadrature formula of the type

$$\int_{-1}^1 f(t)dt = \alpha_{-1} \int_{-1}^{-1/2} f(t)dt + \alpha_0 f(0) + \alpha_1 \int_{1/2}^1 f(t)dt + E(f)$$

having maximum degree of exactness d. What is the value of d?

3. Let $E(f) = \int_a^b f(x)dx - \sum_{k=1}^n w_k f(x_k)$ be the error of a quadrature rule having degree of exactness d. Show that none of the Peano kernels $K_0, K_1, \ldots, K_{d-1}$ can be definite.

4. (a) Use the method of undetermined coefficients to construct a quadrature formula of the type

$$\int_0^1 f(x)dx = af(0) + bf(1) + cf''(\gamma) + E(f)$$

   having maximum degree of exactness d, the variables being a, b, c, and γ.

   (b) Show that the Peano kernel K_d for the formula obtained in (a) is definite, and hence express the remainder in the form $E(f) = e_{d+1} f^{(d+1)}(\xi)$, $0 < \xi < 1$.