Numerical Analysis Assignment #6

1. (a) Suppose the function $f(x) = \ln(2+x)$, $-1 \le x \le 1$, is interpolated by a polynomial p_n of degree $\le n$ at the Chebyshev points $x_k = \cos\left(\frac{2k+1}{2n+2}\pi\right)$, $k = 0, 1, \ldots, n$. Derive a bound for the maximum error $\|f - p_n\|_\infty = \max_{-1\le x\le 1} |f(x) - p_n(x)|$.

   (b) Compare the result of (a) with a bound for $\|f - t_n\|_\infty$, where $t_n(x) = f(0) + \frac{f'(0)}{1!}x + \cdots + \frac{f^{(n)}(0)}{n!}x^n$ is the nth-degree Taylor polynomial of f.

2. Let $T_k(x)$ denote the Chebyshev polynomial of degree k. Clearly, $T_n(T_m(x))$ is a polynomial of degree $n \cdot m$. Identify it.