Numerical Analysis **Assignment 7P#**

1. Let $S_1^0(\Delta)$ be the class of piecewise linear functions relative to the subdivision $a = x_1 < x_2 < \cdots < x_n = b$. Let $\|g\|_\infty = \max\limits_{a \le x \le b} |g(x)|$, and denote by $s_1(g;\cdot)$ the piecewise linear interpolant (from $S_1^0(\Delta)$) to g.
   (a) Show: $\|s_1(g;\cdot)\|_\infty \le \|g\|_\infty$ for any $g \in C[a,b]$.
   (b) Show: $\|f - s_1(f;\cdot)\|_\infty \le 2\|f - s\|_\infty$ for any $s \in S_1^0$, $f \in C[a,b]$. {*Hint:* Use additivity of $s_1(f;\cdot)$ with respect to f.}
   (c) Interpret the result in (b) when s is the best uniform spline approximant to f.

2. From (1.6) with $n = 3$ we know that

$$f'(x_0) = [x_0, x_1]f + (x_0 - x_1)[x_0, x_1, x_2]f + (x_0 - x_1)(x_0 - x_2)[x_0, x_1, x_2, x_3]f + e$$

   where $e = (x_0 - x_1)(x_0 - x_2)(x_0 - x_3)\frac{f^{(4)}(\xi)}{4!}$. Apply this to

$$x_0 = 0, x_1 = \frac{1}{8}, x_2 = \frac{1}{4}, x_3 = \frac{1}{2}$$

   and express $f'(0)$ as a linear combination of $f_k = f(x_k)$, $k = 0, 1, 2, 3$, and e. Also estimate the error e in terms of $M_4 = \max\limits_{0 \le x \le \frac{1}{2}} \left|f^{(4)}(x)\right|$.