Numerical Analysis **Answers to Assignment #6**

1. (a) For $n+1$ distinct nodes x_ν,

Use Newton's formula, the coefficient of $n+1$ item is

$$[x_0, x_1, ..., x_n]f$$

Hence by Lagrange interpolation formula, the coefficient of $n+1$ item is

$$\sum_{\nu=0}^{n} \frac{f(x_\nu)}{\prod_{\mu \neq \nu}(x_\nu - x_\mu)}$$

because the nodes are distinct, so the coefficient of $n+1$ items are the same.

(b) Use the conclusion of (a),

$$[x_0, x_1, ..., x_n](f \cdot g_j) = \sum_{\nu=0}^{n} \frac{f \cdot g(x_\nu)}{\prod_{\mu \neq \nu}(x_\nu - x_\mu)}$$

$$= \sum_{\nu=0}^{j-1} \frac{f(x_\nu) \cdot (x_\nu - x_j)}{\prod_{\mu \neq \nu}(x_\nu - x_\mu)} + \frac{f(x_\nu) \cdot (x_j - x_j)}{\prod_{\mu \neq \nu}(x_\nu - x_\mu)} + \sum_{\nu=j+1}^{n} \frac{f(x_\nu) \cdot (x_\nu - x_j)}{\prod_{\mu \neq \nu}(x_\nu - x_\mu)}$$

$$= \sum_{\nu=0}^{j-1} \frac{f(x_\nu) \cdot (x_\nu - x_j)}{\prod_{\mu \neq \nu}(x_\nu - x_\mu)} + 0 + \sum_{\nu=j+1}^{n} \frac{f(x_\nu) \cdot (x_\nu - x_j)}{\prod_{\mu \neq \nu}(x_\nu - x_\mu)}$$

$$= \sum_{\nu=0}^{j-1} \frac{f(x_\nu)}{\prod_{\mu \neq \nu, \mu \neq j}(x_\nu - x_\mu)} + \sum_{\nu=j+1}^{n} \frac{f(x_\nu)}{\prod_{\mu \neq \nu, \mu \neq j}(x_\nu - x_\mu)}$$

$$= \sum_{\nu=0}^{n} \frac{f(x_\nu)}{\prod_{\mu \neq \nu, \mu \neq j}(x_\nu - x_\mu)}$$

$$= [x_0, x_1, ..., x_{j-1}, x_{j+1}, ..., x_n]f$$

where $g_j(x) = x - x_j$.

2. Computer the fifth divided difference $[0, 1, 1, 1, 2, 2]f$ when $f(x) = x^7$

x	$f(x)$					
0	0					
1	1	1				
1	1	7	6			
1	1	7	21	15		
2	128	127	120	99	42	
2	128	448	321	201	102	30

then

$$p(x) = x + 6x(x-1) + 15x(x-1)^2 + 42x(x-1)^3 + 30x(x-1)^3(x-2)$$

The error term

$$\frac{f^{[5]}(x)}{5!}x(x-1)^3(x-2)^2 \qquad 0 \le \xi \le 2$$

where

$$\frac{f^{[5]}(\xi)}{5!} = [0, 1, 1, 1, 2, 2]f = 30$$

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times \xi^2}{5!} = 30$$

$$\xi = \sqrt{\frac{10}{7}} = 1.1952286$$