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SEPARATING LINEAR MAPS OF CONTINUOUS FIELDS OF BANACH SPACES*

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In this paper, we give a complete description of the structure of separating linear maps between continuous fields of Banach spaces. Some automatic continuity results are obtained.

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1. Introduction

Let T be a locally compact Hausdorff space, called *base space*. Suppose for each t in T there is a (real or complex) Banach space E_t. A *vector field* x is an element in the product space $\prod_{t\in T} E_t$, that is, $x(t) \in E_t$, for all $t \in T$.

Definition 1.1 ([5, 3]). *A continuous field* $\mathcal{E} = (T, \{E_t\}, \mathcal{A})$ *of Banach spaces* over a locally compact space T is a family $\{E_t\}_{t\in T}$ of Banach spaces, with a set $\mathcal{A}$ of vector fields, satisfying the following conditions.

(i) $\mathcal{A}$ is a vector subspace of $\prod_{t\in T} E_t$.
(ii) For every t in T, the set of all $x(t)$ with x in $\mathcal{A}$ is dense in E_t.
(iii) For every x in $\mathcal{A}$, the function $t \mapsto \|x(t)\|$ is continuous on T and vanishes at infinity.

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(iv) Let x be a vector field. Suppose for every t in T and every $\epsilon > 0$, there is a neighborhood U of t and a y in $\mathcal{A}$ such that $\|x(s) - y(s)\| < \epsilon$ for all s in U. Then $x \in \mathcal{A}$.

Elements in $\mathcal{A}$ are called *continuous vector fields*.

When all E_t equal to a fixed Banach space E, and $\mathcal{A}$ consists of all continuous functions from T into E vanishing at infinity, we call $\mathcal{E}$ a *constant field*. In this case, we write $\mathcal{A} = C_0(T, E)$, or $\mathcal{A} = C(T, E)$ when T is compact, as usual.

It is not difficult to see that $\mathcal{A}$ becomes a Banach space under the norm $\|x\| = \sup_{t\in T} \|x(t)\|$. If g is a bounded continuous scalar-valued function on T, and $x \in \mathcal{A}$, then $t \mapsto g(t)x(t)$ defines a continuous vector field gx on T. The set of all $x(t)$ with x in $\mathcal{A}$ coincides with E_t for every t in T. Moreover, for any distinct points s, t in T and any α in E_s and β in E_t, there is a continuous vector field x such that $x(s) = \alpha$ and $x(t) = \beta$ (see, e.g., [5, 12]).

A map $\theta : \mathcal{A} \to \mathcal{B}$ is called a *homomorphism between two continuous fields of Banach spaces* $(X, \{E_x\}_x, \mathcal{A})$ and $(Y, \{F_y\}, \mathcal{B})$ if there is a map $\varphi : Y \to X$ and a linear map $H_y : E_{\varphi(y)} \to F_y$ for each y in Y such that

$$\theta(f)(y) = H_y(f(\varphi(y))), \quad \text{for all } f \in \mathcal{A}, \text{ for all } y \in Y. \tag{1.1}$$

A map θ is said to be *separating* (or *strictly separating* as in [1]) if

$$\|f(x)\|\|g(x)\| = 0, \text{ for all } x \in X, \quad \text{implies} \quad \|\theta(f)(y)\|\|\theta(g)(y)\| = 0, \text{ for all } y \in Y.$$

The study of when a separating linear map is a homomorphism has been the focus of much research in the past. For example, in [10], Jarosz gives a complete description of an unbounded separating linear map $\theta : C(X) \to C(Y)$, where X, Y are compact Hausdorff spaces, and this is extended to locally compact spaces in [7, 11]. On the other hand, Jamison and Rajagopalan [9] show that every *bounded* separating linear map $\theta : C(X, E) \to C(Y, F)$ between continuous vector valued function spaces carries a standard form (1.1). Chan [2] extends this to *bounded* separating linear maps between two function modules.

In this paper, we present a complete description of separating linear maps $\theta : \mathcal{A} \to \mathcal{B}$ between continuous fields of Banach spaces $(X, \{E_x\}_x, \mathcal{A})$ and $(Y, \{F_y\}, \mathcal{B})$ on locally compact Hausdorff base spaces. Essentially, these maps carry the standard form (1.1). In case θ is bijective, and both θ and θ^{-1} are separating, we shall see that $\varphi : Y \to X$ is a homeomorphism. Moreover, θ, as well as the fiber linear maps H_y, is automatically bounded in many situations. Our results unify and extend those shown in [9, 10, 2, 7, 11, 1, 8].

Another example of continuous fields of Banach spaces comes from *Banach bundles*. (The readers are referred to [4, 6] for the definitions.) For a Banach bundle $\xi = (p, E, T)$, define $\Gamma_0(\xi)$ to be the set of all continuous cross sections of ξ which vanishes at infinity. In this case, we write $\mathcal{E} = (T, \{E_t\}, \Gamma_0(\xi))$. It is not difficult to see that $\Gamma_0(\xi)$ satisfies the conditions (i), (iii), (iv) in Definition 1.1. We refer to Appendix C in [6] where it is shown that if T is locally compact, then for any point

x in E there is a continuous cross section f such that $f(p(x)) = x$. Thus, condition (ii) follows. Therefore, all results in this paper apply to Banach bundles. For further development in this line, readers are referred to [13].

2. The results

For a locally compact Hausdorff space X, we write

$$X_\infty = X \cup \{\infty\},$$

for its one-point compactification. If X is already compact, then the point ∞ at infinity is an isolated point in X_∞. Moreover, we identify

$$C_0(X) = \{f \in C(X_\infty) : f(\infty) = 0\},$$

and other similar spaces for those of continuous functions on X vanishing at infinity. For a continuous field $(X, \{E_x\}_x, \mathcal{A})$ of Banach spaces, set for each x in X the sets

$$\begin{aligned} I_x &= \{f \in \mathcal{A} : f \text{ vanishes in a neighborhood in } X_\infty \text{ of } x\}, \\ M_x &= \{f \in \mathcal{A} : f(x) = 0\}. \end{aligned}$$

Theorem 2.1. *Let $\theta : \mathcal{A} \to \mathcal{B}$ be a separating linear map between continuous fields of Banach spaces $(X, \{E_x\}, \mathcal{A})$, $(Y, \{F_y\}, \mathcal{B})$ over locally compact Hausdorff spaces X, Y, respectively. Set*

$$Y_0 = \bigcap \{\ker \theta(f) : f \in \mathcal{A}\}.$$

Then, $\infty \in Y_0$ is compact and there is a continuous map $\varphi : Y \setminus Y_0 \to X_\infty$ such that

$$\theta(I_{\varphi(y)}) \subseteq I_y, \qquad \text{for all } y \in Y \setminus Y_0.$$

Set

$$\begin{aligned} Y_1 &= \{y \in Y \setminus Y_0 : \theta(M_{\varphi(y)}) \subseteq M_y\}, \\ Y_2 &= \{y \in Y \setminus Y_0 : \theta(M_{\varphi(y)}) \nsubseteq M_y\}. \end{aligned}$$

Then there is a linear map $H_y : E_{\varphi(y)} \to F_y$ for each y in Y_1 such that

$$\theta(f)(y) = H_y(f(\varphi(y))), \qquad \text{for all } y \in Y_1.$$

The exceptional set Y_2 is open in Y_∞, and $\varphi(Y_2)$ consists of finitely many non-isolated points in X_∞.

Moreover, θ is bounded if and only if $Y_2 = \emptyset$ and all H_y are bounded. In this case,

$$\|\theta\| = \sup_{y \in Y} \|H_y\|.$$

We divide the proof into several lemmas as in [10, 11]. Clearly, Y_0 is compact and contains ∞. For each y in $Y \setminus Y_0$, let

$$Z_y = \{x \in X_\infty : \theta(I_x) \subseteq I_y\}.$$

Lemma 2.1. *Z_y is a singleton, for all y in $Y \setminus Y_0$.*

Proof. Suppose on the contrary that $Z_y = \emptyset$ for some y in $Y \setminus Y_0$. Then for each x in X_∞ there is an f_x in I_x vanishing in a compact neighborhood U_x of x such that $\theta(f_x) \notin I_y$. By compactness,

$$X_\infty = U_{x_0} \cup U_{x_1} \cup \cdots \cup U_{x_n}$$

for some points $x_0 = \infty$, $x_1, \ldots, x_n$ in X_∞. Let

$$1 = h_0 + h_1 + \cdots + h_n$$

be a continuous partition of unity such that h_i vanishes outside U_{x_i} for $i = 0, 1, \ldots, n$. For any g in $\mathcal{A}$, the separating property of θ implies that the product of the norm functions of $\theta(h_i g)$ and $\theta(f_{x_i})$ is always zero, and then

$$\theta(f_{x_i}) \notin I_y \quad \text{implies} \quad \theta(h_i g)(y) = 0, \quad i = 0, 1, \ldots, n.$$

Hence, $\theta(g)(y) = 0$ for all $g \in \mathcal{A}$. This gives a contradiction that $y \in Y_0$.

Next let x_1, x_2 be distinct points in Z_y. In other words, $\theta(I_{x_i}) \subseteq I_y$ for $i = 1, 2$. Choose compact neighborhoods V, U of x_1 in X_∞ such that V is contained in the interior of U, and $x_2 \notin U$. Let $g \in C(X_\infty)$ such that $g = 1$ on V and $g = 0$ outside U. Then for all f in $\mathcal{A}$, the facts $(1-g)f \in I_{x_1}$ and $gf \in I_{x_2}$ ensure that $\theta(f) \in I_y$. In particular, $y \in Y_0$, a contradiction again. □

Define a map $\varphi : Y \setminus Y_0 \to X_\infty$ by

$$Z_y = \{\varphi(y)\}.$$

In other words, $\theta(I_{\varphi(y)}) \subseteq I_y$, or

$$f \in I_{\varphi(y)} \quad \text{implies} \quad \theta(f) \in I_y, \qquad \text{for all } y \in Y \setminus Y_0. \tag{2.1}$$

Lemma 2.2. *$\varphi : Y \setminus Y_0 \to X_\infty$ is continuous.*

Proof. Suppose $y_\lambda \to y$ in $Y \setminus Y_0$, but $x_\lambda = \varphi(y_\lambda) \to x \neq \varphi(y)$. By Lemma 2.1, $\theta(I_x) \nsubseteq I_y$. Let $U_x, U_{\varphi(y)}$ be disjoint compact neighborhoods of $x, \varphi(y)$, respectively. Let $g \in C(X_\infty)$ such that $g = 1$ on U_x and $g = 0$ on $U_{\varphi(y)}$. Since $x_\lambda \to x$, for all f in A, $(1-g)f$ is eventually in I_{x_λ}. Thus, $\theta((1-g)f) \in I_{y_\lambda}$ eventually. By the continuity of the norm function, $\theta((1-g)f)(y) = 0$. On the other hand, $gf \in I_{\varphi(y)}$ implies $\theta(gf) \in I_y$. Hence, $\theta(f)(y) = 0$ for all $f \in \mathcal{A}$. This gives $y \in Y_0$, a contradiction. □

Denote by δ_y the evaluation map at y in Y, i.e.,

$$\delta_y(g) = g(y) \in F_y, \qquad \text{for all } g \in \mathcal{B}.$$

Lemma 2.3. *Let $\{y_n\}$ be a sequence in $Y \setminus Y_0$ such that $\varphi(y_n)$ are distinct points in X_∞. Then*

$$\limsup \|\delta_{y_n} \circ \theta\| < +\infty.$$

Proof. Suppose not, by passing to a subsequence if necessary, we can assume the norm $\|\delta_{y_n} \circ \theta\| > n^4$, and there is an f_n in $\mathcal{A}$ such that $\|f_n\| \leq 1$ and $\|\theta(f_n)(y_n)\| > n^3$, for $n = 1, 2, \ldots$. Let $x_n = \varphi(y_n)$ and V_n, U_n be compact neighborhoods of x_n in X_∞ such that V_n is contained in the interior of U_n, and $U_n \cap U_m = \emptyset$, for distinct $n, m = 1, 2, \ldots$. Let $g_n \in C(X_\infty)$ such that $g_n = 1$ on V_n and $g_n = 0$ outside U_n for $n = 1, 2, \ldots$. Observe

$$\begin{aligned}\theta(f_n)(y_n) &= \theta(g_n f_n)(y_n) + \theta((1 - g_n)f_n)(y_n) \\ &= \theta(g_n f_n)(y_n), \quad \text{as } (1 - g_n)f_n \in I_{x_n}.\end{aligned}$$

So we can assume f_n is supported in U_n, for $n = 1, 2, \ldots$. Let

$$f = \sum_{n=1}^{\infty} \frac{1}{n^2} f_n \in \mathcal{A}.$$

Since $n^2 f - f_n \in I_{x_n}$, we have $n^2\theta(f)(y_n) = \theta(f_n)(y_n)$ by (2.1), and thus $\|\theta(f)(y_n)\| > n$, for $n = 1, 2, \ldots$. As $\theta(f)$ in $\mathcal{B}$ has a bounded norm, we arrive at a contradiction. □

Set

$$\begin{aligned}Y_1 &= \{y \in Y \setminus Y_0 : \theta(M_{\varphi(y)}) \subseteq M_y\}, \\ Y_2 &= \{y \in Y \setminus Y_0 : \theta(M_{\varphi(y)}) \nsubseteq M_y\}.\end{aligned}$$

Lemma 2.4. *$\varphi(Y_2)$ is a finite set consisting of non-isolated points in X_∞.*

Proof. Let $x = \varphi(y)$ with y in Y_2. Then by (2.1) we have

$$\theta(I_x) \subseteq I_y \quad \text{but} \quad \theta(M_x) \nsubseteq M_y.$$

Since, by Uryshons Lemma, I_x is dense in M_x, this implies the linear operator $\delta_y \circ \theta$ is unbounded. By Lemma 2.3, we can only have finitely many of such x's. So $\varphi(Y_2)$ is a finite set. Moreover, if x is an isolated point in X_∞ then $I_x = M_x$, and thus $x \notin \varphi(Y_2)$. □

Proof. [Proof of Theorem 2.1] Let $y \in Y_1$, we have $\theta(M_{\varphi(y)}) \subseteq M_y$. Hence, there is a linear operator $H_y : E_{\varphi(y)} \to F_y$ such that

$$\theta(f)(y) = H_y(f(\varphi(y))), \quad \text{for all } f \in \mathcal{A}. \tag{2.2}$$

Next we want to see that Y_2 is open, or equivalently, $Y_0 \cup Y_1$ is closed in Y_∞. Let $y_\lambda \to y$ with y_λ in $Y_0 \cup Y_1$. We want to show that $y \in Y_0 \cup Y_1$. Since Y_0 is compact, we might assume $y_\lambda \in Y_1$ for all λ.

In case there is any subnet of $\{\varphi(y_\lambda)\}$ consisting of only finitely many points, we can assume $\varphi(y_\lambda) = \varphi(y)$ for all λ. Then for all f in $\mathcal{A}$, $f(\varphi(y)) = 0$ implies

$f(\varphi(y_\lambda)) = 0$, and thus $\theta(f)(y_\lambda) = 0$ for all λ by (2.2). By continuity, $\theta(f)(y) = 0$. Consequently, $\theta(M_{\varphi(y)}) \subseteq M_y$, and thus $y \in Y_0 \cup Y_1$.

In the other case, every subnet of $\{\varphi(y_\lambda)\}$ contains infinitely many points. Lemma 2.3 asserts that $M = \limsup \|H_{y_\lambda}\| < +\infty$. This gives

$$\|\theta(f)(y)\| = \lim \|\theta(f)(y_\lambda)\| = \lim \|H_{y_\lambda}(f(\varphi(y_\lambda)))\| \leq M\|f(\varphi(y))\|.$$

Thus, if $f(\varphi(y)) = 0$ we have $\theta(f)(y) = 0$. Consequently, $y \in Y_0 \cup Y_1$.

Observe that the boundedness of θ implies $Y_2 = \emptyset$. Moreover,

$$\begin{aligned}\|\theta\| &= \sup\{\|\theta(f)\| : f \in \mathcal{A} \text{ with } \|f\| = 1\}\\ &= \sup\{\|H_y(f(\varphi(y)))\| : f \in \mathcal{A} \text{ with } \|f\| = 1, y \in Y_1\}\\ &\leq \sup\{\|H_y\| : y \in Y_1\}.\end{aligned}$$

The reverse inequality is plain.

Finally, we suppose $Y_2 = \emptyset$ and all H_y are bounded. We claim that $\sup \|H_y\| < +\infty$. Otherwise, there is a sequence $\{y_n\}$ in Y_1 such that $\lim_{n\to\infty} \|H_{y_n}\| = +\infty$. By Lemma 2.3, we can assume all $\varphi(y_n) = x$ in X. Let $e \in E_x$ and $f \in \mathcal{A}$ such that $f(x) = e$. Then

$$\|H_{y_n}(e)\| = \|\theta(f)(y_n)\| \leq \|\theta(f)\|, \quad n = 1, 2, \ldots.$$

It follows from the uniform boundedness principle that $\sup \|H_{y_n}\| < +\infty$, a contradiction. It is now obvious that θ is bounded. □

The following extends the results for constant fields shown in [1, 8].

Theorem 2.2. *Let $(X, \{E_x\}, \mathcal{A})$, $(Y, \{F_y\}, \mathcal{B})$ be continuous fields of Banach spaces over locally compact Hausdorff spaces X, Y, respectively. Let $\theta : \mathcal{A} \to \mathcal{B}$ be a bijective linear map such that both θ and its inverse θ^{-1} are separating. Then there is a homeomorphism φ from Y onto X, and a bijective linear operator $H_y : E_{\varphi(y)} \to F_y$ for each y in Y such that*

$$\theta(f)(y) = H_y(f(\varphi(y))), \quad \textit{for all } f \in \mathcal{A}, \textit{ for all } y \in Y.$$

Moreover, at most finitely many H_y are unbounded, and this can happen only when y is an isolated point in Y. In particular, if X (or Y) contains no isolated point then θ is automatically bounded.

Proof. Since θ is onto, we have $Y_0 = \{\infty\}$. Because both θ, θ^{-1} are separating, there are continuous maps $\varphi : Y \to X_\infty$ and $\psi : X \to Y_\infty$ such that

$$\theta(I_{\varphi(y)}) \subseteq I_y \quad \text{and} \quad \theta^{-1}(I_{\psi(x)}) \subseteq I_x, \quad \text{for all } x \in X, y \in Y.$$

In case $\psi(x) \neq \infty$, this gives

$$\theta(I_{\varphi(\psi(x))}) \subseteq I_{\psi(x)} \subseteq \theta(I_x),$$

or

$$I_{\varphi(\psi(x))} \subseteq I_x.$$

It follows $\varphi(\psi(x)) = x$ for all x in X with $\psi(x) \neq \infty$. Similarly, we will have $\psi(\varphi(y)) = y$ for all y in Y with $\varphi(y) \neq \infty$. Set $X_3 = X \setminus \psi^{-1}(\infty)$ and $Y_3 = Y \setminus \varphi^{-1}(\infty)$. It is then easy to see that $\varphi = \psi^{-1}$ induces a homeomorphism from Y_3 onto X_3. By the bijectivity of θ, the open sets X_3 and Y_3 contain X_1 and Y_1, respectively.

Next, we want to see that $Y_2 = \emptyset$ and $Y_1 = Y_3 = Y$. Indeed, by Theorem 2.1, $Y_2 \cap Y_3$ is open, and a finite set (as $\varphi(Y_2)$ is). Hence $Y_2 \cap Y_3$ consists of isolated points in Y, and so does $\varphi(Y_2 \cap Y_3)$. It then follows from Lemma 2.4 that $Y_2 \cap Y_3$ is empty. Consequently, $Y_1 = Y_3$ and $\varphi(Y_2) \subseteq \{\infty\}$. Similarly, $X_1 = X_3$ and $\psi(X_2) \subseteq \{\infty\}$. It follows from (2.1) and the injectivity of θ that $\varphi(Y)$, and thus $\varphi(Y_1) = X_1$, is dense in X. As X_1 is closed in X, we see that $X = X_1$ and thus $X_2 = \emptyset$. Correspondingly, $Y = Y_1$ and $Y_2 = \emptyset$. It turns out that φ is a homeomorphism from Y onto X with inverse ψ.

Now $Y = Y_1$ and $X = X_1$ implies that both θ and θ^{-1} can be written as homomorphisms of continuous fields of Banach spaces:

$$\theta(f)(y) = H_y(f(\varphi(y))), \quad \text{for all } f \in \mathcal{A}, \text{ for all } y \in Y,$$

$$\theta^{-1}(g)(x) = T_x(g(\psi(x))), \quad \text{for all } g \in \mathcal{B}, \text{ for all } x \in X.$$

It is easy to see that the linear map $H_y : E_{\varphi(y)} \mapsto F_y$ has an inverse $T_{\varphi(y)}$ for every y in Y, and thus it is bijective.

By Lemma 2.3, at most finitely many H_y are unbounded. Let y be a non-isolated point in Y. We will show that the linear map H_y is bounded. Suppose not, then for each $n = 1, 2, \ldots$ there is an f_n in $\mathcal{A}$ of norm one such that $\|\theta(f_n)(y)\| = \|H_y(f_n(\varphi(y)))\| > n^4$. By the continuity of the norm of $\theta(f_n)$, there are all distinct points y_n of Y in a neighborhood of y such that $\|\theta(f_n)(y_n)\| > n^3$. Let $x_n = \varphi(y_n)$ in X for $n = 1, 2, \ldots$. Since φ is a homeomorphism, we can also assume that all x_n are distinct with disjoint compact neighbourhoods U_n. By multiplying with a norm one continuous scalar function, we can assume each f_n is supported in U_n. Let $f = \sum_n \frac{1}{n^2} f_n$ in $\mathcal{A}$. Since $n^2 f - f_n \in I_{x_n}$, we have $n^2\theta(f)(y_n) = \theta(f_n)(y_n)$ and thus $\|\theta(f)(y_n)\| > n$ for $n = 1, 2, \ldots$. This contradiction tells us that H_y is bounded for all non-isolated y in Y_1.

The last assertion follows from Theorem 2.1, and we have $\|\theta\| = \sup \|H_y\| < +\infty$. □

Remark 2.1.

(1) Unlike the scalar case, if any fiber E_x of the continuous field of Banach spaces $(X, \{E_x\}, \mathcal{A})$ is of infinite dimension, some H_y can be unbounded in Theorem 2.2. This happens even for the constant fields based on compact spaces. See Example 2.4 in [8].

(2) There is a counterexample in ([8], Example 3.1) of a continuous bijective separating linear map between constant fields based on nonhomeomorphic compact

spaces, whose inverse is not separating. So the biseparating assumption in Theorem 2.2 cannot be dropped.

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