ZERO PRODUCT PRESERVING MAPS OF OPERATOR VALUED FUNCTIONS

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Abstract. Let X, Y be locally compact Hausdorff spaces and $\mathcal{M}, \mathcal{N}$ be Banach algebras. Let $\theta : C_0(X, \mathcal{M}) \to C_0(Y, \mathcal{N})$ be a zero-product preserving bounded linear map with dense range. We show that θ is given by a continuous field of algebra homomorphisms from $\mathcal{M}$ into $\mathcal{N}$ if $\mathcal{N}$ is irreducible. As corollaries, such a surjective θ arises from an algebra homomorphism, provided that $\mathcal{M}$ is a W*-algebra and $\mathcal{N}$ is a semi-simple Banach algebra, or both $\mathcal{M}$ and $\mathcal{N}$ are C*-algebras.

1. Introduction

Let X be a locally compact Hausdorff space. Denote by $X_\infty = X \cup \{\infty\}$ the one-point compactification of X. In case X is already compact, ∞ is an isolated point in X_∞. For a real or complex Banach algebra $\mathcal{M}$, let $C_0(X, \mathcal{M}) = \{f \in C(X, \mathcal{M}) : f(\infty) = 0\}$ be the Banach algebra of all continuous vector-valued functions from X into $\mathcal{M}$ vanishing at infinity. Note that $C_0(X, \mathcal{M})$ is isometrically and algebraically isomorphic to the (projective) tensor product $C_0(X) \otimes \mathcal{M}$.

In this paper, we shall study those bounded linear maps θ from $C_0(X, \mathcal{M})$ into another such algebra $C_0(Y, \mathcal{N})$ preserving zero products. Namely, $fg = 0$ implies $\theta(f)\theta(g) = 0$. In other words,

$$f(x)g(x) = 0 \text{ in } \mathcal{M} \text{ for all } x \in X \implies \theta(f)(y)\theta(g)(y) = 0 \text{ in } \mathcal{N} \text{ for all } y \in Y.$$

For example, let $\sigma : Y \to X$ be a continuous function, let h be a uniformly bounded norm continuous function from Y into the center of $\mathcal{N}$, and let φ be a uniformly bounded SOT continuous function from Y into $B(\mathcal{M}, \mathcal{N})$ such that each $\varphi_y = \varphi(y)$ is an algebra homomorphism. Then

$$\theta(f)(y) = h(y)\varphi_y\big(f(\sigma(y))\big) \tag{1.1}$$

defines a zero-product preserving bounded linear map from $C_0(X, \mathcal{M})$ into $C_0(Y, \mathcal{N})$. In particular, $\theta = h\varphi$ for a bounded central element h in the algebra $C(Y, \mathcal{N})$ and an

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algebra homomorphism φ from $C_0(X, \mathcal{M})$ into $C_0(Y, \mathcal{N})$. We will investigate when zero product preserving bounded linear maps arise in this way.

For the scalar case, every zero-product preserving bounded linear map θ from $C_0(X)$ into $C_0(Y)$ is of the expected form (1.1) [13, 11, 14]. Recall that a subalgebra $\mathcal{S}$ of the algebra $B(E)$ of all bounded linear operators on a Banach space E is said to be *standard* if $\mathcal{S}$ contains all continuous finite rank operators. Using an interesting geometric approach, Araujo and Jarosz [2] showed that when X, Y are realcompact and $\mathcal{M}$ and $\mathcal{N}$ are standard operator algebras, every bijective linear map from $C(X, \mathcal{M})$ onto $C(Y, \mathcal{N})$ preserving zero products in both directions is in the form of (1.1). However, in the non-bijective case it becomes a very difficult task without assuming continuity. Even discontinuous algebra homomorphisms have complicated structure ([15, 19]). Finally, readers are referred to [1, 12, 6, 21] for problems of similar interests.

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2. Results

A linear map θ from $C_0(X, \mathcal{M})$ into $C_0(Y, \mathcal{N})$ is said to be *strictly separating* if

$$\|f(x)\|\|g(x)\| = 0 \text{ for all } x \in X \quad \Longrightarrow \quad \|Tf(y)\|\|Tg(y)\| = 0 \text{ for all } y \in Y.$$

Denote by $\operatorname{coz}(f) = \{x \in X : f(x) \neq 0\}$ the *cozero set* of an f in $C_0(X, \mathcal{M})$. Then θ is strictly separating if and only if it preserves the disjointness of cozeroes. We note that a subset U of X is the cozero of a continuous function in $C_0(X, \mathcal{M})$ if and only if U is σ-compact and open. For any σ-compact open subset U of X, denote by $C_0(U, \mathcal{M})$ the subalgebra of all f in $C_0(X, \mathcal{M})$ with $\operatorname{coz}(f) \subseteq U$.

Recall that a representation $\pi : \mathcal{N} \to B(E)$ of a Banach algebra $\mathcal{N}$ is said to be *faithful* if the kernel of π is $\{0\}$. We call π an *irreducible representation* of $\mathcal{N}$ if there is no proper linear subspace F of the Banach space E such that $\pi(\mathcal{N})F \subseteq F$. It amounts to say that for each nonzero vector e in E, the linear subspace $\pi(\mathcal{N})e$ is the whole of E. Every irreducible representation of a Banach algebra is automatically bounded [15]. A Banach algebra $\mathcal{N}$ is said to be *irreducible* if it has a faithful irreducible representation $\pi : \mathcal{N} \to B(E)$.

Theorem 1. *Let X and Y be locally compact Hausdorff spaces. Let $\mathcal{M}$ and $\mathcal{N}$ be Banach algebras such that $\mathcal{N}$ is irreducible, and let θ be a continuous zero-product preserving linear map from $C_0(X, \mathcal{M})$ into $C_0(Y, \mathcal{N})$ with dense range. Then θ is strictly separating.*

Indeed, there exists a continuous map $\sigma : Y \to X$, and for each y in Y a bounded zero-product preserving linear map $H_y : \mathcal{M} \to \mathcal{N}$ with dense range such that

$$\theta(f)(y) = H_y(f(\sigma(y))) \quad \textit{for all } f \in C_0(X, \mathcal{M}) \textit{ and } y \in Y.$$

Moreover, the correspondence $y \mapsto H_y$ defines a uniformly bounded map $H : Y \to B(\mathcal{M}, \mathcal{N})$ continuous in the strong operator topology.

Proof. Let $\pi : \mathcal{N} \to B(E)$ be a faithful irreducible representation of $\mathcal{N}$. Composing θ with π, we can assume that $\mathcal{N}$ is an irreducible subalgebra of $B(E)$ and θ is again bounded and zero-product preserving with dense range.

Fix y in Y, and denote by

$$S_y = \Big\{ x \in X_\infty : \text{for all } \sigma\text{-compact open neighborhood } U \text{ of } x, \\ \text{there is an } f \text{ in } C_0(U, \mathcal{M}) \text{ such that } \theta(f)(y) \neq 0, \\ \text{that is, } \theta|_{C_0(U,\mathcal{M})} \text{ is not trivial at } y \Big\}.$$

Claim 1. $S_y \neq \varnothing$.

Suppose not, and for each x in X_∞, there is a σ-compact open neighborhood U_x of x such that $\theta|_{C_0(U_x,\mathcal{M})}$ is trivial at y. Write

$$X_\infty = U_0 \cup U_1 \cup \cdots \cup U_n$$

for $x_0 = \infty$, and some $x_1, \ldots, x_n$ in X, with a σ-compact open neighborhood U_i for $i = 0, 1, \ldots, n$, respectively. Let

$$1 = f_0 + f_1 + \cdots + f_n$$

be a continuous partition of the unity such that $\operatorname{coz} f_i \subseteq U_i$ for $i = 0, 1, \ldots, n$. Then for all f in $C_0(X, \mathcal{M})$,

$$\theta(f) = \theta(f_0 f + f_1 f + \cdots + f_n f) = 0,$$

since $\operatorname{coz}(f_i f) \subseteq U_i$ for each $i = 0, 1, \ldots, n$. This is impossible.

Claim 2. $x_1, x_2 \in S_y \implies x_1 = x_2$.

Suppose $x_2 \neq x_1 \neq \infty$. Let U_1 and U_2 be disjoint σ-compact open neighborhoods of x_1 and x_2, respectively. We can assume that $\infty \notin U_1$. Since

$$f_1 f_2 = f_2 f_1 = 0 \quad \text{for all } f_i \in C_0(U_i, \mathcal{M}),\ \ i = 1, 2,$$

we have

$$\theta(f_1)\theta(f_2) = \theta(f_2)\theta(f_1) = 0 \quad \text{in } C_0(Y, \mathcal{N}).$$

Let E_1 be the intersection of the kernels of all $\theta(f_1)(y)$ with f_1 in $C_0(U_1, \mathcal{M})$. Because both $\theta|_{C_0(U_1,\mathcal{M})}$ and $\theta|_{C_0(U_2,\mathcal{M})}$ are not trivial at y, we have E_1 is a proper subspace of E, that is, $\{0\} \neq E_1 \neq E$.

Let V be a nonempty open set in Y such that $\overline{V} \subseteq U_1$. Let g be in $C_0(X)$ such that $\operatorname{coz} g \subseteq U_1$ and $g|_V = 1$. For each f in $C_0(X, \mathcal{M})$, write

$$f = fg + f(1-g).$$

Since $\operatorname{coz}(fg) \subseteq U_1$, we have

$$\theta(fg)(y)|_{E_1} = 0.$$

Hence

$$\theta(f)(y)|_{E_1} = \theta(f(1-g))(y)|_{E_1}.$$

For any k in $C_0(X, \mathcal{M})$ with $\operatorname{coz} k \subseteq V$, we have $k(f(1-g)) = 0$. This implies

$$\theta(k)(y)\theta(f)(y)|_{E_1} = \theta(k)(y)\theta(f(1-g))(y)|_{E_1} = 0 \quad \text{for all } f \in C_0(X, \mathcal{M}).$$

However, $\{\theta(f)(y) : f \in C_0(X, \mathcal{M})\}$ is dense in $\mathcal{N}$, which is irreducible on E. Therefore,

$$\theta(k)(y) = 0 \quad \text{for all } k \in C_0(X, \mathcal{M}) \text{ with } \operatorname{coz} k \subseteq V.$$

Since V is an arbitrary nonempty open set with closure contained in U_1, we have

$$\theta(k)(y) = 0 \quad \text{for all } k \in C_0(U_1, \mathcal{M}).$$

This conflict establishes Claim 2.

By Claims 1 and 2, S_y is a singleton.

Claim 3. If $S_y = \{x\}$ then

$$f(x) = 0 \quad \Longrightarrow \quad \theta(f)(y) = 0.$$

By Urysohn's Lemma, we can assume f vanishes in a neighborhood of x. Now $x \notin \overline{\operatorname{coz} f}$, which is compact in X_∞. For each x' in $\overline{\operatorname{coz} f}$, there is a σ-compact open neighborhood U' of x' such that $\theta|_{C_0(U', \mathcal{M})}$ is trivial at y. By a compactness argument as the one proving Claim 1, we see that $\theta(f)(y) = 0$.

It follows from Claim 3 that $S_y \neq \{\infty\}$ for all y in Y since θ has dense range. Denote by $\sigma(y) = x$ if $S_y = \{x\}$. Then there is a linear map $H_y : \mathcal{M} \to \mathcal{N}$ such that

$$\theta(f)(y) = H_y\left(f(\sigma(y))\right) \quad \text{for all } f \in C_0(X, \mathcal{M}) \text{ and } y \in Y.$$

In particular, θ is strictly separating.

The rest of the proof follows in a straightforward manner, or one can quote the standard results about strictly separating maps in [6, 12]. □

The following lemma might be known, although we do not find a proof from the literature. Remark that it is shown in [17] every non-zero Banach algebra homomorphism from $B(H)$ into $B(K)$ is injective if both H and K are separable Hilbert spaces. However, there is an example in [18] of a non-zero homomorphism from $B(H)$ into $B(H)$ with compact operators as its kernel, where H is inseparable. Moreover, it is known that every irreducible representation of a Banach algebra is norm continuous [15] and every algebra isomorphism between C*-algebras is a $*$-isomorphism [20, Theorem 4.1.20].

Lemma 2. *Let H, K be real or complex Hilbert spaces of arbitrary dimension. Let $B(H)$ and $B(K)$ be the algebras of all bounded linear operators on H and K, respectively. Then every surjective algebra homomorphism from $B(H)$ onto $B(K)$ is an isomorphism.*

Proof. The case is trivial when H is of finite dimension since $B(H)$ is then a simple algebra. Suppose the (Hilbert space) dimension of H is an infinite cardinal number $\aleph_H$. For each infinite cardinal number $\aleph \leq \aleph_H$, let $I_\aleph$ be the closed two-sided ideal of $B(H)$ consisting of operators T such that all closed subspaces contained in the range of T is of dimension less than $\aleph$. In case H is separable, $I_{\aleph_H} = \mathcal{K}(H)$, the ideal of compact operators on H. In general, as indicated in [5] that $I_{\aleph_H}$ is the largest two-sided ideal of $B(H)$. In fact, every closed two-sided ideal of $B(H)$ is in the form of $I_\aleph$ for some $\aleph \leq \aleph_H$ [9, Section 17].

Let θ be an algebra homomorphism from $B(H)$ onto $B(K)$. Then the kernel I of θ is a closed two-sided ideal of $B(H)$. Since the quotient algebra $B(H)/I$ is isomorphic to $B(K)$, there is an e in $B(H)$ such that $(e + I)B(H)(e + I) = eB(H)e + I$ is of one dimension modulo I. Assume I is nonzero. Let $\aleph$ be the infinite cardinal number such that $I = I_\aleph$. Then the range of e contains a closed subspace of dimension $\aleph$. By halving this subspace into two each of dimension $\aleph$, we see that $eB(H)e$ contains two elements linear independent modulo $I_\aleph$, a contradiction. This completes our proof. ∎

Corollary 3. *Let X, Y be locally compact Hausdorff spaces. Let $\mathcal{M}, \mathcal{N}$ be either the Banach algebras $B(H), B(K)$ of all bounded operators or $\mathcal{K}(H), \mathcal{K}(K)$ of compact operators on real or complex Hilbert spaces H, K, respectively. Let $\theta : C_0(X, \mathcal{M}) \to C_0(Y, \mathcal{N})$ be a continuous surjective zero-product preserving linear map. Then there exist a continuous function σ from Y into X, a continuous scalar function h on Y, and a SOT continuous map $y \mapsto S_y$ from Y into $B(K, H)$ such that S_y is invertible and*

$$\theta(f)(y) = h(y){S_y}^{-1} f(\sigma(y)) S_y, \quad \forall f \in C(X, \mathcal{M}), \forall y \in Y. \tag{2.1}$$

Proof. It follows from Theorem 1 that for each fixed y in Y, θ induces a bounded zero-product preserving linear map $H(y)$ from $\mathcal{M}$ onto $\mathcal{N}$. By either [10, Theorem 2.1] or [7, Corollary 3.2], $H(y)$ is a scalar multiple of a bounded algebra homomorphism from $\mathcal{M}$ onto $\mathcal{N}$. Since $\mathcal{K}(H)$ is simple, this algebra homomorphism is indeed an isomorphism if $\mathcal{M}$ and $\mathcal{N}$ are $\mathcal{K}(H)$ and $\mathcal{K}(K)$, respectively. On the other hand, by Lemma 2 the algebra homomorphism above is again an isomorphism in case $\mathcal{M}$ and $\mathcal{N}$ are $B(H)$ and $B(K)$, respectively. Thus, by either [3, Theorem 4] or [8, Corollary 3.2], there exist a scalar $h(y)$ and a bounded invertible operator S_y on K to implement (2.1). It is then routine to check the continuity of h and the map $y \mapsto S_y$. ∎

The following corollary holds, for example, when $\mathcal{M}$ is a W^*-algebra, or a unital C^*-algebra of real rank zero [4].

Corollary 4. *Let X and Y be locally compact Hausdorff spaces such that X is compact. Let $\mathcal{M}$ be a unital Banach algebra such that the subalgebra of $\mathcal{M}$ generated by its idempotents is norm dense in $\mathcal{M}$, and let $\mathcal{N}$ be a semi-simple Banach algebra. Let θ be a continuous zero-product preserving linear map from $C(X,\mathcal{M})$ into $C_0(Y,\mathcal{N})$ with dense range. Then $\theta(1)$ is in the center of $C_0(Y,\mathcal{N})$, and*

$$\theta(1)\theta(fg) = \theta(f)\theta(g) \quad \text{for all } f,g \in C(X,\mathcal{M}). \tag{2.2}$$

Suppose, in addition, that Y is compact and $\mathcal{N}$ is unital. If $\theta(1)$ is invertible or θ is surjective, then $\theta = \theta(1)\varphi$ for an algebra homomorphism φ.

Proof. Let $\pi : \mathcal{N} \to B(E)$ be an irreducible representation of $\mathcal{N}$. Then $\theta_\pi = \pi \circ \theta$ is again a continuous zero-product preserving linear map from $C(X,\mathcal{M})$ into $C_0(Y,\pi(\mathcal{N}))$ with dense range. By Theorem 1, we find that θ_π carries a weighted composition operator form

$$\theta_\pi(f)(y) = H_y\big(f(\sigma(y))\big) \quad \text{for all } f \in C(X,\mathcal{M}) \text{ and } y \in Y.$$

In particular, each H_y is a continuous zero-product preserving linear map from $\mathcal{M}$ into $\pi(\mathcal{N})$ with dense range.

By results in [10] (see also [7]), for each y in Y we have $\theta_\pi(1)(y) = H_y(1)$ is in the center of $\mathcal{N}$ and

$$H_y(1)H_y(ab) = H_y(a)H_y(b) \quad \text{for all } a,b \in \mathcal{M}.$$

Hence

$$\pi\left(\theta(1)\theta(f) - \theta(f)\theta(1)\right) = 0$$

and

$$\pi\left(\theta(1)\theta(fg) - \theta(f)\theta(g)\right) = 0$$

for all f,g in $C(X,\mathcal{M})$. Being semi-simple, $\mathcal{N}$ has a faithful family of irreducible representations. Thus $\theta(1)$ is in the center of $C_0(Y,\mathcal{N})$ and (2.2) holds.

Now, we assume that Y is compact and $\mathcal{N}$ is unital. If θ is surjective, $1 = \theta(f)$ for some f in $C(X,\mathcal{M})$. It follows from $\theta(1)\theta(f^2) = \theta(f)^2 = 1$ that $\theta(1)$ is invertible. Assume $\theta(1)$ is invertible. Then $\theta(1)^{-1}\theta$ is again a bounded zero-product preserving linear map with dense range, and sends 1 to 1. Suppose now $\theta(1) = 1$. Then (2.2) ensures that θ is an algebra homomorphism. □

A recent result in [7] states that every surjective zero-product preserving bounded linear map θ between unital C*-algebras is a product $\theta = \theta(1)\varphi$ of the invertible central element $\theta(1)$ and an algebra homomorphism φ. Since $C(X,\mathcal{A})$ (resp. $C(Y,\mathcal{B})$) is $*$-isomorphic to the (projective) tensor product $C(X)\otimes\mathcal{A}$ (resp. $C(Y)\otimes\mathcal{B}$) as C^*-algebras (see, e.g., [16]), we have the following

Corollary 5. *Let X and Y be compact Hausdorff spaces, and $\mathcal{A},\mathcal{B}$ be unital C*-algebras. Let θ be a continuous zero-product preserving linear map from $C(X,\mathcal{A})$ onto $C(Y,\mathcal{B})$. Then $\theta(1)$ is an invertible element in the center of $C(Y,\mathcal{B})$, and $\theta=\theta(1)\varphi$ for an algebra homomorphism φ.*

The following example shows that the irreducibility condition on $\mathcal{N}$ cannot be dropped in Theorem 1, and the map θ in the Corollaries 4 and 5 cannot be written as a weighted composition operator in the form of (1.1) in general.

Example 6. Let $X=\{0\}$ and $\mathcal{M}=\mathbb{C}\oplus\mathbb{C}$ be the two-dimensional C*-algebra, and let $Y=\{1,2\}$ and $\mathcal{N}=\mathbb{C}$ be the one-dimensional C*-algebra. Define $\theta: C(X,\mathcal{M})\to C(Y,\mathcal{N})$ by $\theta(a\oplus b)=g$ with $g(1)=a$ and $g(2)=b$. Then θ is bijective and preserves zero products in both directions.

Remark that $\theta: C(X,\mathcal{M})\to C(Y,\mathcal{N})$ satisfies the condition stated in Theorem 1. In fact, let $h_1(a\oplus b)=a$ and $h_2(a\oplus b)=b$ be the canonical projection of $\mathbb{C}\oplus\mathbb{C}$ onto its summands, and set $\sigma(1)=\sigma(2)=0$. Then

$$\theta(f)(y)=h_y(f(\sigma(y))),\quad \forall f\in C(X,\mathcal{M}),\forall y\in Y.$$

However, $\mathcal{M}$ is not irreducible and $T^{-1}: C(Y,\mathcal{N})\to C(X,\mathcal{M})$ does not carry a weighted composition operator form. Note also that X and Y are not homeomorphic although both $C(X,\mathcal{M})$ and $C(Y,\mathcal{N})$ are isomorphic to $\mathbb{C}\oplus\mathbb{C}$ as C*-algebras and θ implements an algebra isomorphism between them.

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