A Locally Conservative Streamline Method for a Model Two-Phase Flow Problem in a One-Dimensional Porous Medium

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Abstract

Motivated by possible generalizations to more complex multiphase multicomponent systems in higher dimensions, we develop a numerical approximation for a system of two conservation laws in one space dimension modeling two-phase flow in a porous medium. The method is based on tracing streamlines, so it is stable independent of any CFL constraint. The main difficulty is that it is not possible to trace individual streamlines independently. We approximate streamline tracing using local mass conservation principles and self-consistency. The two-phase flow problem is governed by a system of equations representing mass conservation of each phase, so there are two local mass conservation principles. Our numerical method respects both of these conservation principles over the computational mesh (i.e., locally), and so is a fully conservative streamline method. We present numerical results that demonstrate the ability of the method to handle problems with shocks and rarefactions, and to do so with very coarse spatial grids and time steps larger than the CFL limit.

Keywords: Computational physics and chemistry, Partial differential equations, advection-diffusion, characteristics, streamlines.