Regularization Techniques for the Method of Fundamental Solutions

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The Method of Fundamental Solutions (MFS), which can be regarded a meshless version of the (indirect) Boundary Element Method, exhibits a lot of numerical advantages. First, it is a truly meshless technique, i.e. it requires neither domain nor boundary element structure; only a finite set of unstructured points is necessary. Next, it requires no sophisticated difference schemes to approximate the appearing derivatives (e.g. upwind discretization of the convective derivative in transport problems); also, there is no need to use carefully defined elements for discretizing the pressure and the velocities when solving the Stokes problem etc. However, a scattered data interpolation is often required. Using the popular globally supported radial basis functions (e.g. multiquadrics, inverse multiquadrics, thin plate or polyharmonic splines), this leads to a subproblem with a high computational cost, if the number of the base points of the interpolation is large. To reduce the computational cost, special techniques (e.g. domain decomposition) have to be applied. On the other hand, in the traditional version of the Method of Fundamental Solutions, some source points lying outside of the domain of the partial differential equation have to be introduced, due to the singularity of the applied fundamental solution at the origin. If these source points are located far from the boundary, the resulting system of algebraic equations becomes extremely ill-conditioned, which can destroy the numerical efficiency.

In this paper, a special kind of regularization techniques is applied. The partial differential operator appearing in the original problem is approximated by a higher order singularly perturbed operator with a carefully chosen scaling parameter. This new partial differential operator is (at least) of fourth order, and its fundamental solution is continuous at the origin. Using the fundamental solution of this new operator instead of the original one, the source and the collocation points are allowed to coincide. In this way, the use of extremely ill-conditioned systems can be avoided. Thus, a classical second order problem (e.g. the Poisson equation, the modified Helmholtz equation, as well as the convection-diffusion equation or also the classical Stokes equations) can be converted to another problem in which the operator is a fourth order multi-elliptic operator, but has a continuous fundamental solution. In some cases, the solution of the resulting linear equations can be completely avoided by using multi-elliptic interpolation and quadtree-based multigrid tools. These techniques are useful also in creating particular solutions. The applicability of the approach is demonstrated via several numerical examples.

Keywords: method of fundamental solutions, regularization, multi-elliptic operator, multi-elliptic interpolation, Stokes equations, quadtree, multigrid.