Energy Derivative Valuation Using Radial Basis Function

Ryosuke Fujiwara, Naohiro Sekiya, and Eisuke Kita

Graduate School of Information Science, Nagoya University
Nagoya 464-8601
Japan
kita@is.nagoya-u.ac.jp

A derivative contract is a financial contract whose value is derived from the values of one or more underlying assets, reference rates, or indices of asset values or reference rates. In this study, we will focus on the energy derivative. The energy derivative is a derivative contract in which the underlying asset is based on energy products including oil, natural gas and electricity, which trade either on an exchange or over-the-counter.

When the derivative price V depends on the energy spot price S and the time t, the price V should satisfy the differential equation:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+\alpha\left(\mu-\lambda-\ln S\right)S\frac{\partial V}{\partial S}-rV=0 \tag{1}$$

The expiration condition of the European future option price is given as follows

$$\begin{aligned} V(F,T) &= \max(F(T)-E,0) \qquad \text{(for Call)} \\ V(F,T) &= \max(E-F(T),0) \qquad \text{(for Put)} \end{aligned}$$

where the parameter E is the strike price on the expiration date T. The function F is given as

$$\begin{aligned} F\left(t,T\right) \;=\; \exp\Bigg[& e^{-\alpha(T-t)}\ln S+\left(1-e^{-\alpha(T-t)}\right)\left(\mu-\lambda-\frac{\sigma^2}{2\alpha}\right) \\ & +\frac{\sigma^2}{4\alpha}\left(1-e^{-2\alpha(T-t)}\right)\Bigg] \end{aligned} \tag{2}$$

Use of equation (2) gives the following equation from equation (1)

$$\frac{\partial V}{\partial t}+\frac{e^{-2\alpha(T-t)}}{2}\sigma^2 F^2\frac{\partial^2 V}{\partial F^2}-rV=0 \tag{3}$$

The above equation is discretized with Crank-Nicolson method and the radial bases function approximation $V \simeq \sum \lambda_j^t \phi_j$. Finally, we have

$$HV^{t+\Delta t}=GV^t \tag{4}$$

The symbols H and G are differential operators.

Equation (4) is solved backward from the expiration date T to the date of purchase in order to valuate the derivative price.

Keywords: Energy Derivative, Radial Basis Function, Meshless Method.