The Inverse Determination of the Thermal Contact Resistance Between Components of Unidirectionally Reinforced Composite

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In this work a boundary collocation Trefftz method has been presented for two-dimensional steady-state heat conduction analysis of fiber reinforced composites. The heat flow in a composite is a complex phenomenon and can only be understood through a proper micromechanical analysis. The solution of the heat conduction problem is usually based on the solution of heat transfer equation at microstructure level in repeated element of an array[1]. The calculations are carried out for a composite with the fibers arranged in a matrix in a regular manner by square grid (Fig. 1a) with imperfect thermal contact between the constituents. The heat conduction problem of composites with imperfect thermal contact was considered in papers[2,3]. The thermal conductivities of constituents (fibers λ_f and matrix λ_m, $F=\lambda_f/\lambda_m$) and the volume fraction of fibers $\varphi=\pi E^2/4$, where $E=a/b$, are known.

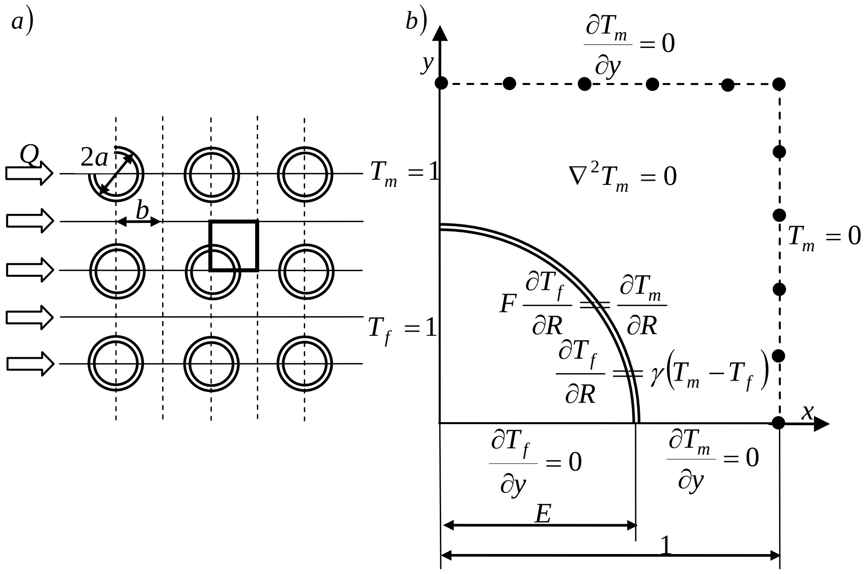

Figure 1 A unidirectional reinforced fibrous composite with fiber arrangement according to a square array for the imperfect thermal contact between fiber and matrix: *a)* general view, *b)* formulation boundary value problem in a repeated element

The purpose of this paper is to propose an analytic-numerical algorithm for determination of the thermal contact resistance γ in unidirectionally reinforced composite. Such a problem can be treated as a kind of the inverse heat conduction problem in inhomogenous material.
Because the algorithm for the inverse problem is in some sense based on the solution of the direct problem (Fig.1b) we begin with the boundary Trefftz method to determine the temperature field:

$$T_f = 1 + \sum_{k=1}^{N} w_k R^{(2k-1)} \cos[(2k-1)\theta] \tag{1}$$

$$T_m = 1 + \sum_{k=1}^{N} \frac{w_k}{2} \left[\begin{array}{l} \left(F + 1 + \frac{(2k-1)}{\gamma E} \right) R^{(2k-1)} + \\ - \left(F - 1 - \frac{(2k-1)}{\gamma E} \right) \frac{E^{2(2k-1)}}{R^{(2k-1)}} \end{array} \right] \cos[(2k-1)\theta] \tag{2}$$

and the effective thermal conductivity [1-3]:

$$\frac{\lambda_z}{\lambda_m} = \sum_{k=1}^{N} \frac{w_k}{2} (-1)^k \left[\left(F + 1 + \frac{(2k-1)}{\gamma E} \right) + \left(F - 1 - \frac{(2k-1)}{\gamma E} \right) E^{2(2k-1)} \right] \tag{3}$$

where T_f and T_m are non-dimensional temperatures in the matrix and in the fiber respectively, w_k are unknown coefficients determined from the boundary conditions.
For the boundary conditions:

$$T_m = 0 \quad \text{for} \quad x = 1 \tag{4a}$$

$$\frac{\partial T_m}{\partial y} = 0 \quad \text{for} \quad y = 1 \tag{4b}$$

and for the additional condition determined by:

a. a known value of temperature in $N3$ points located on the matrix:

$$T_m(R,\theta) = T_i \,.\, i = 1,\ldots,N3 \quad \text{for} \quad R > E \,. \tag{5a}$$

b. a known value of temperature in $N3$ points located on the upper edge:

$$T_m(R,\theta) = T_i \,.\, i = 1,\ldots,N3 \quad \text{for} \quad Y = 1 \tag{5a}$$

c. a known value of the effective thermal conductivity (3):

$$\frac{\lambda_z}{\lambda_m}(F,\gamma,E) = \lambda_{eff\,.} \tag{5c}$$

the resistance number and the unknown coefficients w_k**,** k=1,…, N are calculated.

For the substitution $w_{N+1} = \gamma$ from the collocation [4] of the boundary condition (4a) in $N1$ points on the right edge and the boundary condition (4b) in $N2$ points on upper edge of the considered region and from condition (5) we obtain a system of Ne=$N1$+ $N2$+ $N3$ nonlinear equations G with Nu= $N+1$ with the unknowns w_k which is solved in least square sense:

$$\delta(w_1,\ldots,w_{N+1}) = \sum_{i=1}^{NR} [G_i(w_1,\ldots,w_{N+1})]^2 \,, \tag{6}$$

$$\frac{\partial \delta}{\partial w_k} = 2\sum_{i=1}^{NR} [G_i(w_1,\ldots,w_{N+1})] \frac{\partial G_i}{\partial w_k} = 0, \qquad k = 1,\ldots,N+1 \,.$$

For solving this problem the Levenberg-Marquadt iteration method is used:

$$\sum_{l=1}^{N+1} A'_{k,l} \cdot \delta w_l = b_k \,, \quad k = 1,2,..,N+1 \tag{7}$$

where: $b_k = -\dfrac{\partial \delta}{\partial w_k}$, $A_{k,l} = \dfrac{\partial^2 \delta}{\partial w_k \partial w_l}$, $A'_{k,l} = \begin{cases} A_{k,l} \cdot (1+\lambda) & k = l \\ A_{k,l} & k \neq l \end{cases}$ is used.

The study compared three different cases of the additional conditions (5). In addition to determining the resistance number γ, the impact of the number of collocation points ($N1$, $N2$) on the quality of the results was examined. Because an iterative method was used the convergence had to be considered**.** The method was not always quickly convergent.

Keywords: Trefftz Method, Levenberg-Marquadt Method, Thermal Contact Resistance, Unidirectionally Reinforced Composite

References

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