

# **The Modified Collocation Trefftz Method and Exponentially Convergent Scalar Homotopy Algorithm for the Inverse Boundary Determination Problem**

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In this paper, the modified collocation Trefftz method (MCTM) and the exponentially convergent scalar homotopy algorithm (ECSHA) are adopted to analyze the inverse boundary determination problem governed by the biharmonic equation. The position for part of boundary with given boundary condition is unknown and the position for the rest of boundary with additionally specified boundary conditions is given. Since the spatial position for portion of boundary is not given a priori, it is extremely difficult to solve the boundary determination problem by any numerical scheme. In order to stably solve the boundary detection problem, the MCTM, one kind of boundary-type meshless methods, will be adopted in this study, since it can avoid the generation of mesh grid and numerical integration. In the boundary determination problem governed by the biharmonic equation, the numerical solution of MCTM is expressed as linear combination of the T-complete function. When this problem is considered by MCTM, a system of nonlinear algebraic equations will be formed and solved by ECSHA which will converge exponentially. The unknown coefficients in MCTM and the position of the unknown boundary can be found simultaneously by the evolutionary process of ECSHA. Some numerical examples will be provided to demonstrate the ability and accuracy of the proposed scheme. Besides, the stability of the proposed meshless method will be proven by adding some noise into the boundary conditions.

**Keywords:** modified collocation Trefftz method, exponentially convergent scalar homotopy algorithm, boundary determination problem, biharmonic equation, boundary-type meshless method.