

## **The Least Squares Trefftz Method with External Source for the Eigenfrequencies of Waveguides**

**Yan-Chen Liu**

**Department of Harbor and River Engineering & Computation and Simulation Center  
National Taiwan Ocean University  
2 Pei-Ning Road, Keelung  
Taiwan  
[cmfan@ntou.edu.tw](mailto:cmfan@ntou.edu.tw)**

**Joint work with: Chia-Ming Fan, Hsin-Fang Chan, Sung-Shan Hsiao**

In this paper, the least squares Trefftz method (LSTM) with external sources is proposed to analyze the eigenfrequencies of waveguide. The LSTM, the combination of the conventional Trefftz method and the least squares method, can stabilize the numerical scheme and obtain highly accurate numerical solutions for Helmholtz problem. In LSTM, the numerical solution is expressed as the linear combination of T-complete functions and the number of equations is larger than the number of knowns to avoid the ill-conditioning problem in conventional Trefftz method. The determination of the eigenfrequencies of waveguide, governed by the two-dimensional Helmholtz equation, will form an eigenproblem. Using external sources, the eigenproblem will be converted to an inhomogeneous Helmholtz problem. Then the proposed LSTM will be adopted to analyze the inhomogeneous problem. By recording the responses from a series of tests, the eigenfrequencies of waveguides can be found. In comparing with the technique of singular value decomposition and the method of determinant search, the computational cost of the proposed boundary-type meshless scheme for dealing with the eigenproblems can be evidently reduced. Several numerical examples will be provided to validate the proposed LSTM. Both of the transverse magnetic wave and the transverse electric wave will be examined by the proposed meshless method to demonstrate the capability and robustness of the present method.

**Keywords:** least squares Trefftz method, external source, eigenfrequencies, waveguide, boundary-type meshless scheme.