Fast Chebyshev pseudospectral Poisson solver for all kinds of boundary conditions via diagonalization

Poisson equation for a simple 2D or 3D rectangular domain is frequently encountered in scientific and engineering problems, and a fast Poisson solver would be needed. Poisson equation using Chebyshev pseudospectral method is usually solved in two ways. One is discretizing the Laplace operator to a large matrix based on tensor product of collocation derivative matrix and then solved by sparse solvers. Since the formed matrix is very large even for moderate resolution, it is very time consuming to solve and suffers insufficient computer memory frequently. The other is using iterative method which is also time consuming when resolution is high and sometimes has poor convergence when pre-conditioning is not applied. Actually, an ultra-fast Poisson solver via diagonalization using eigenvectors of collocation derivative matrix has been developed as early as 1987. However, it only works with Dirichlet boundary conditions exclusively, and no Neumann or Robin boundary condition is allowed. This pitfall has severely restricted its usage. The current work removed this serious constraint by adopting penalty method to manipulate boundary conditions of all kinds, and it computes basically as fast as the original one with Dirichlet boundary condition only. The spectral accuracy is obtained when resolution is enough to resolve solution structure and a penalty weight is selected appropriately. This penalty method can be also extended to solve Helmholtz equation and cylindrical Poisson equation.

Keywords: Poisson equation, Chebyshev pseudospectral method, collocation derivative matrix, diagonalization.