The singularity method: On the motion of a rotating sphere in unsteady Stokes flows

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The singularities of potential flow such as the sources, sinks, vortices and doublets have been well known for more than one hundred years. For steady Stokes flows, the singular solution also had been derived since the work of Lorentz (1897). For the following development of the singularity method, a family of fundamental solutions called Stokeslet, Rotlet, Stokeslet doublet, Stresslet, and so on was investigated, which had been further employed to construct exact solutions to exterior and interior steady Stokes flow problems, but non-stationary Stokes flow problems are seldom presented. In this paper, the fundamental solution of an unsteady Rotlet (also called a couplet) for the unsteady Stokes equation is derived. The surface force and torque exerted on a fluid sphere centered at the pole of the singularity is also presented. The couple on a sphere starting to rotate with an arbitrary differentiable angular velocity $\boldsymbol{\omega}_{\mathbf{p}}(t)$ in low Reynolds number flows is calculated by applying the fundamental solution of an unsteady Rotlet. The results are compared with exact solutions by Feuillebois and Lasek (1977). Because the feature of the singularity method for steady Stokes flow is that the force and torque are given by the total strength of the singularities, it is also calculated and compared for unsteady Stokes flow.

Keywords: Singularity method, unsteady Stokes flow, Rotlet.