

Condition Number and the Related Mathematical Study on Boundary Meshless Methods for the Laplace Equation in an Exterior Unbounded Domain

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The method of fundamental solution (MFS) can easily and efficiently solve a boundary value problem of the Laplace equation in an exterior unbounded domain. The numerical solution obtained by the MFS is accurate although the corresponding matrix equation is ill-conditioned. The reduction of the condition number of the matrix is however required to stabilize the numerical solution. The MFS proposed in this study uses proper basis functions that strictly satisfy a condition at infinity, and the corresponding matrix is approximately obtained and has a lower condition number by way of the modified Trefftz method (MTM).

The mathematical expressions of the corresponding condition numbers are derived. The solvability of the MTM and our method is mathematically proven. Thereby, the optimal parameter minimizing the condition number can explicitly be given in a mathematical form. Numerical experiments show that the condition number is reduced and that the numerical solution by the proposed method is more accurate than the one by the conventional method.

Keywords: Condition number, Exterior unbounded domain, Laplace equation, Method of fundamental solution, Modified Trefftz method.