Radial Basis Function Methods Incorporated with a Manifold-Based Exponentially Convergent Algorithm for Solving Partial Differential Equations

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In this study, a Manifold-Based Exponentially Convergent Algorithm (MBECA) combined with radial basis function (RBF) methods for solving partial differential equations (PDEs) is proposed. RBF methods have shown great advantages of the grid free property of the methods, and they are becoming a viable choice as a method for the numerical solution of PDEs. We first employ RBF methods to discretize PDEs into nonlinear algebraic equations. Then, the MBECA is adopted to solve the nonlinear algebraic equations. The MBECA is based on the construction of a space-time manifold to transform a vector function into a time-dependent scalar function by introducing a fictitious time variable. Taking the advantages of the time-dependent scalar function, the proposed novel MBECA does not need to calculate the inverse of the Jacobian matrix and has a great advantage that it is exponentially convergent. Illustration examples demonstrate that the MBECA can dramatically improve the convergence and has a greater performance than other nonlinear equation solvers. Several numerical examples including elliptic type PDEs are examined and compared to exact solutions. Results obtained show that the proposed MBECA combined with radial basis function (RBF) methods can significantly improve the accuracy as well as the convergence for solving PDEs.

Keywords: Manifold-Based Exponentially Convergent Algorithm (MBECA), Radial Basis Function (RBF), Jacobian matrix, Partial Differential Equations (PDEs), Non-Linear Algebraic Equations.