Boundary Element solution of Stokes Nano-flow between curved surfaces with linear and nonlinear boundary condition

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ABSTRACT

The numerical simulation of Micro/Nano fluid flow through the solution of governing equations based on continuum models has to be done under the consideration of appropriate slip boundary conditions to account for the velocity jump at the solid-fluid interface. The liner slip boundary condition states a relation between the tangential shear rate and the fluid-wall velocity differences and has been successfully used in reproducing the characteristics of many types of flows (e.g. slit flows, rotating curved mixers, microbearings, among others), where the shear rate at solid-fluid interfaces remains linear because of the geometry smoothness. Despite this, there are some situations for which this linear dependency fails leading to unrealistic behaviour since the slip length becomes nonlinear and singular like is the case of spreading liquids and corner flows, where a variation in the slip length is suitable due to the presence of regions of higher shear stress and shear rate. The previous behaviour was summarized in an expression for the slip condition at a solid-liquid interface establishing the variation in the slip length in terms of the square root of the tangential shear rate as well. Moreover incorrect flow behaviours has been previously reported in some numerical works dealing with flow fields confinement by curved surfaces, due to wrong evaluation of the local tangential shear rate at the wall surfaces.

In this work a boundary integral equation formulation for Stokes slip flow, based on the normal and tangential projection of the Green's integral representational formulae for the Stokes velocity field, which directly incorporates into the integral equations the local tangential shear rate at the wall surfaces, is presented. The resulting boundary integral equation for the slip flow is function of the tangential projection of the shear rate, allowing the evaluation of the nonlinear terms that appear due to the inclusion of the universal slip flow boundary condition. The Boundary Element Method (BEM) is employed to solve the resulting projections of the integral equations coupled with the slip condition as function of surface traction in the tangential direction. The equation system is evaluated iteratively turning the nonlinear term into a nonhomogeneous constant vector by using results from previous iteration.

The proposed BEM formulation is used to simulate flow between curved rotating geometries: i.e. concentric and eccentric Couette and single rotor mixers. The numerical results obtained for the concentric Couette mixer are validated with the corresponding analytical solutions under linear and nonlinear boundary condition, showing excellent agreements. Results obtained in this work extend the use of BEM for the study of microfluid flow, allowing the developed of more complex Micro/Nano fluidic applications.

With the idea of comparing the proposed approach with more simpler alternative scheme, the case of concentric Couette flow with linear and nonlinear boundary condition was also solved with the Method of Fundamental Solutions (MFS) showing that for simple geometric this approach is very efficient.