Miscellaneous open problems in the Regular Boundary Collocation approach

Andrzej P. Zielinski,
Institute of Machine Design,
Cracow University of Technology
Al. Jana Pawla II 37, 31-864 POLAND,
e-mail: apz@mech.pk.edu.pl

Certain characteristic aspects of the generalized Trefftz method have been analysed. In this method the trial functions identically fulfill the governing differential equations of the considered Boundary Value Problem (BVP) inside the whole region Ω including its boundary Γ. To approximate well the solution, these trial functional sets (called Trefftz- or T-functions) should be complete. The above requirements are fulfilled by at least two types of functional systems: the general solutions of homogeneous governing equations of the problem, as well as its fundamental solutions with singularities outside the investigated region Ω. In the latter case, the location of singularities presents an open problem, which is discussed in detail and illustrated by numerical examples. In both above cases the solution inside Ω is obtained as a sum of regular functions with unknown coefficients. The coefficients are determined from the regular boundary integral equations resulting from the boundary and connectivity conditions of the problem.

In large majority of cases the above integrals are calculated numerically. It has be proved, that *the numerical integral fit of the approximate solution is a specific version of collocation in the integral control points with integration weights. Therefore, the notion of boundary collocation can be considered as more general*. In this case the situating of control points and the choice of collocation weights is an open question of the considered problem. It has been discussed in detail.

Usually the boundary Γ consists of segments. From the physical point of view, certain sub-regions of Ω, and thus certain segments, can be more important than the other. Hence, we can influence the accuracy of solution in the important regions by introducing additional weights in the segments of Γ near these regions. The additional weights are also important in connectivity of different regions, because of difference between the essential and natural continuity conditions. This open problem has been discussed and illustrated by numerical examples.

The solution of the BVP defined in a large and complex area Ω often requires division of Ω into subregions. This process is called substructuring. Several typical methods of sub-structuring are presented on numerical examples. Their advantages and drawback are discussed. It should be underlined that they still require careful investigations but, so far, the results are very encouraging.

There are several directions in which the generalized Trefftz method (or the Regular Boundary Collocation approach) can be developed. In opinion of the author, the most promising are structural and multidisciplinary optimization, inverse formulations of mechanics and certain nonlinear problems. The current developments in these directions are presented and discussed.