Joint International Workshop on Trefftz Method VI and Method of Fundamental Solutions II

Department of Applied Mathematics
National Sun Yat-sen University
Kaohsiung, Taiwan

March 15-18, 2011

Edited by
Z. C. Li, T. T. Lu, A. H.-D. Cheng
D. L. Young, J. T. Chen, C. S. Chen
and Y. T. Lee

Content

Preface

History of Trefftz VI

The International Workshop on Trefftz methods (Trefftz VI) in Kaohsiung 2011 is the 6th in a series of workshops organised every three years. The former conferences were held in Cracow (Poland 1996), Sintra (Portugal 1999), Exceter (UK 2002), Zilina (Slovakia 2005) and Leuven (Belgium 2008) all in Europe. The presentation of Trefftz method can be traced to 1926 by Trefftz. He proposed a solution with unknown coefficients to satisfy the given governing equation in advance. Then, the unknown coefficients are determined via satisfying boundary conditions. During several decades, many variants of Trefftz methods were developed, e.g. direct and indirect Trefftz formulations. An overview article presented by E. Kita and N. Kamiya in 1995 can be found in *Advances in Engineering Software*. Since the first workshop hosted in 1996, a large number of brilliant works on the Trefftz methodology and its applications for engineering problems can be found. The Trefftz-based methods are powerful and efficient for analyzing engineering problems such as solid mechanics, thermal analysis, bio-mechanics, vibro-acoustics, structural dynamics and optics, etc. The Trefftz VI will provide a platform for people using this method.

Photo from the first conference “Trefftz Method” organized in Cracow in 1996. The persons in the photo are (from left): S. Y. Reutskiy (Ukraine), J. A. Teixeira de Freitas (Portugal), S. Abo Diab (Syria), J. A. Kolodziej (Poland), B. Tirozzi (Italy), H. A. Mang (Austria), E. A. W. Maunder (UK), E. Stein (Germany), Ch. Hochard (France), V. Kompis (Slovakia), A. Wróblewski (Poland), I. Herrera (Mexico), B. Szybinski

(Poland), O. C. Zienkiewicz (UK), R. Piltner (USA), M. H. Zienkiewicz (UK), F. Ihlenburg (USA), J. M. Melenk (Switzerland) and A. P. Zielinski (Poland).

History of MFS II

The first international workshop on the Method of Fundamental Solutions (MFS I) was organized by C. S. Chen, A. Karageorghis and Y.-S. Smyrlis and was held in Ayia Napa, Cyprus from June 11-13, 2007. For the MFS, it was first proposed by V. D. Kupradze and M. A. Alexidze in 1960s. The main idea of MFS is to distribute the singularity outside the domain of the given problem and represents the solution by superimposing the fundamental solutions with unknown coefficients which are determined by satisfying the boundary conditions. From 1970s, the MFS gradually became a useful technique and is used to solve various physical and engineering problems. A number of papers were presented by Mathon, Johnston and Fairweather. In the following two decades, largely due to the effort of Graeme Fairweather and Andreas Karageorghis, the MFS has proven to be an effective method for solving certain classes of homogeneous problems such as inverse problems, free-boundary problems, and time-dependent problems. In 1990s, M. A. Golberg and C. S. Chen extended the MFS to solving inhomogenous problems using the radial basis functions. After all these efforts and success, the MFS has attracted the attention in the science and engineering community. One of the advantages of the MFS over other numerical methods is its simplicity, especially for solving problems in high dimension with irregular domain. Recently, many investigators have focused on developing the regularized techniques in which the fictitious boundary is not required in the formulation of the MFS. The MFS is known by many different names in the literature such as the charge simulation method, the indirect boundary element method, the superposition method, the desingularized method, and the virtual boundary element method, etc. During the past two decades, the meshless methods are getting popular in scientific computing. Among them, the MFS can be considered not only the simplest but also a truly meshless method. Despite all these positive features, there are still limitations and difficulties to be overcome in term of real applications as well as its theoretical foundations. The second MFS workshop will provide a platform for researchers to present some of the recent important development in the MFS.

About the Joint Conference on Trefftz method/MFS

As aforementioned, the two methods individually have a long history and play important roles in the development of computational methods. However, the link between the Trefftz method and the MFS was not noticed in the literature. Until 2007, J. T. Chen and his coworkers published an article in *Computers and Mathematics with*

Applications to discuss the equivalence of the two methods. Later, an EABE 2009 paper by Chen and his coworkers demonstrated the equivalence by considering an annular case. The relationship between MFS and Trefftz method was connected together from the viewpoint of interpolating base. Later, Robert Schaback also found that the behavior of the far-field singularity in the MFS is similar to the Trefftz bases of harmonic polynomials. Regarding to these two topics, many issues, stability and accuracy, are still needed to be further explored. It is a proper time to connect the two methods, Trefftz method and MFS. Nowadays, the two conferences, Trefftz VI and MFS II organized by Z. C. Li, D. L. Young, J. T. Chen, T. T. Lu, A. H.-D. Cheng and C. S. Chen, are held at the same time in NSYSU (Kaohsiung, Taiwan). This is a good opportunity to provide the two groups for sharing their excellent works and studying from each other. We hope that all participants will have a good time during the conference.

The conference contains six plenary talks and seventy-six invited and contributed talks. All the abstracts are contained herein. We would like to acknowledge the great effort on behalf of both the authors and reviewers. Also, we particularly wish to thank the National Science Council (NSC, Taiwan), Mathematics Research Promotion Center (NSC-MRPC), National Sun Yat-sen University (NSYSU), Scientific Formosa INC. and MSV of National Taiwan Ocean University (NTOU) to sponsor the conference.

Taiwan was called as Formosa which means Beautiful Island by the Europeans. This is the first time the two conferences are moved to Asia. Although Taiwan is an island and surrounded by sea, we have more than 200 mountains, our unique geology and topography, countless arresting landscapes and alluring coastal scenes. Besides, in Taiwan, we have world-class natural landscapes (Taroko Gorge), the repository of true Chinese culture (National Palace Museum) which is listed one of the five top museums in the world, the former-three world’s tallest certified building (Taipei 101), and the highest mountain in Northeast Asia (Yushan or called Jade Mountain). We also have 24-hour bookstores, the convenience shops, bustling night markets and the main importance, the friendliest people. All of which make visitors indelible. Finally, welcome to NSYSU and welcome to Formosa, Taiwan.

March, 2011, Taiwan
Chairs of the conference

Organizing Committee

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About NSYSU

Introduction

National Sun Yat-sen University (NSYSU) was founded in 1980 in Kaohsiung and became one of the fastest growing universities in Taiwan and in Asia. NSYSU has endeavored to pursue excellence in teaching and research and has received generous funding from the Ministry of Education under the "Aiming for Top University Plan" (or the ""Five-Year-Five-Billion Project"). With Six college including Liberal Arts, Science, Engineering, Management, Marine Sciences, and Social Sciences, NSYSU offers 20 Bachelor's, 37 Master's and 27 Doctoral Programs. There are currently 9,476 students enrolled and 459 full-time faculty members. Currently, there are nearly 400 International students at NSYSU as the University offers many courses taught in English. Also, the Chinese Language Center provides different levels of courses to improve language and to know the culture.

Besides academic and research prestige, the university campus is situated by both the ocean and the mountain where water sports like surfing and boat sailing by the beach. Interesting species such as macaque monkeys and Cacatus parrots which add multiplex life for faculties and students.

History

In 1923, just two years before his untimely demise, the Founding Father of the Republic of China (ROC), Dr. Sun Yat-sen, established two universities, one civil and one military. The civil university was Kwangtung University (renamed Chungshan University in 1926), and the military university was Whampoa Military Academy. Both univserties generated remarkable contributions and played an important education role in modern Chinese history.

After Chungshan University (or Sun Yat-sen University) was established in Canton, it had to move south due to the Sino-Japanese War. The university moved first to Yunnan, and later to Yuehpei. Finally, in 1945, the university settled back to its original campus in Canton, and schooling continued.

In 1949, the government of the ROC moved to Taiwan. The National Sun Yet-sen University (NSYSU) was meant to be re-established, but the right timing never came along until 1980, when Dr. Huan Li established the university on its current campus at Hsitzu Bay in Kaohsiung. Succeeding presidents Dr. Chin-chi Chao, Dr. Chi-Yuan Lin, Dr. Victor W. Liu, Dr. Chung-cheng Chang and current president Dr. Hung-Duen Yang, all carry the tradition of nurturing the university's growth and development. When it was established, the university had four departments, two graduate institutes, and 189 students. Today, the university has six colleges (Liberal

Arts, Sciences, Engineering, Management, Marine Sciences, and Social Sciences) and over 9300 students, offering 19 undergraduate majors, 34 master programs, and 26 doctorate programs. NSYSU has grown tremendously and recognized as one of Taiwan's important research and learning institution with international distinction.

Blessed with unique historical background, NSYSU enjoys the honor of being a university with outstanding tradition. Infused with the Taiwan Experience after the re-establishment in Kaohsiung, NSYSU is a young and vivacious university with positive outlooks.

Department of Applied Mathematics National Sun Yat-sen University

About Us

Founded in 1987, the Department of Applied Mathematics offers B.Sc., M.Phil. and Ph.D. programs. We have currently 211 undergraduate, 64 master and 27 Ph.D. students. There are now 18 faculty members in the department, divided into 3 divisions: Statistics, Scientific Computing, and Mathematics. Research interests include statistics, scientific computing, nonlinear analysis, discrete mathematics and differential equations. Some faculty members are in the editorial boards of international mathematical journals.

As a research oriented department, we are ranked as one of the leading mathematics departments in Taiwan. In 2002, National Sun Yat-sen University was selected by Ministry of Education as one of the seven research universities in Taiwan, destining for major developments. The Department is equipped with excellent library resources and computer facilities. Many graduates become professionals such as universities professors, school teachers, system analysts, actuarial scientists, financial analysts, … , etc.

College Atrium

The campus is located in the beautiful southern city Kaohsiung, which is the second largest city in Taiwan. Overlooking the Taiwan Strait along with a clean beach, the campus provides unparalleled natural beauty.

The department emphasizes both teaching and research. To better coordinate the research efforts, statistics, scientific computing and mathematics have been identified as our primary research directions:

The statistics research field concentrates on optimal design for regression, financial time series, and applied probability. The recently research work focuses on advanced determinant calculus of D-optimal design for weighted polynomial regression, optimal designs for multi-response non-linear models, estimation of integrated volatility ratio and online monitoring system for high frequency transaction data, some variations of two-person red-and-black game.

The sientific cmputation field studies elliptic boundary value problems with singularity, transport problem, porous media, pattern recognition, polynomial system, matrix computation and computational finance, etc. Beside the traditional finite difference, finite element and finite volume methods, we also use Trefftz and boundary method, method of fundamental solution, radial basis method, combined method and conservative schemes, etc. In addition, we have deep investigation on super convergence of numerical method and effective condition number of linear system.

The Department Library

The mathematics research field concentrates on ring theory, graph theory, operator theory, operator algebra, nonlinear analysis, mathematical programming, optimization problem, inverse nodal and inverse spectral problems, financial mathematics and probability theory.

Research Equipment

The Department is the center of mathematics library of National Science Council in Kaohsiung area. The current, backup issues of journals and online e-journals are fairly complete. There are two computer rooms for graduate and undergraduate students, five multi-media classrooms, equipments for long distance learning and video conferencing respectively. The high-performance computing facility, hardware and software are available. Every graduate student is provided office space.

Online distance course

Education Objectives

Our broad and innovative courses, some with new teaching technologies, provide students the mathematical, statistical and computational skills necessary to pursue advanced study, obtain a career in teaching or in the private and public sectors. Besides, many of our students take interdiscipline programs which let them apply mathematics to various areas.

About NTOU

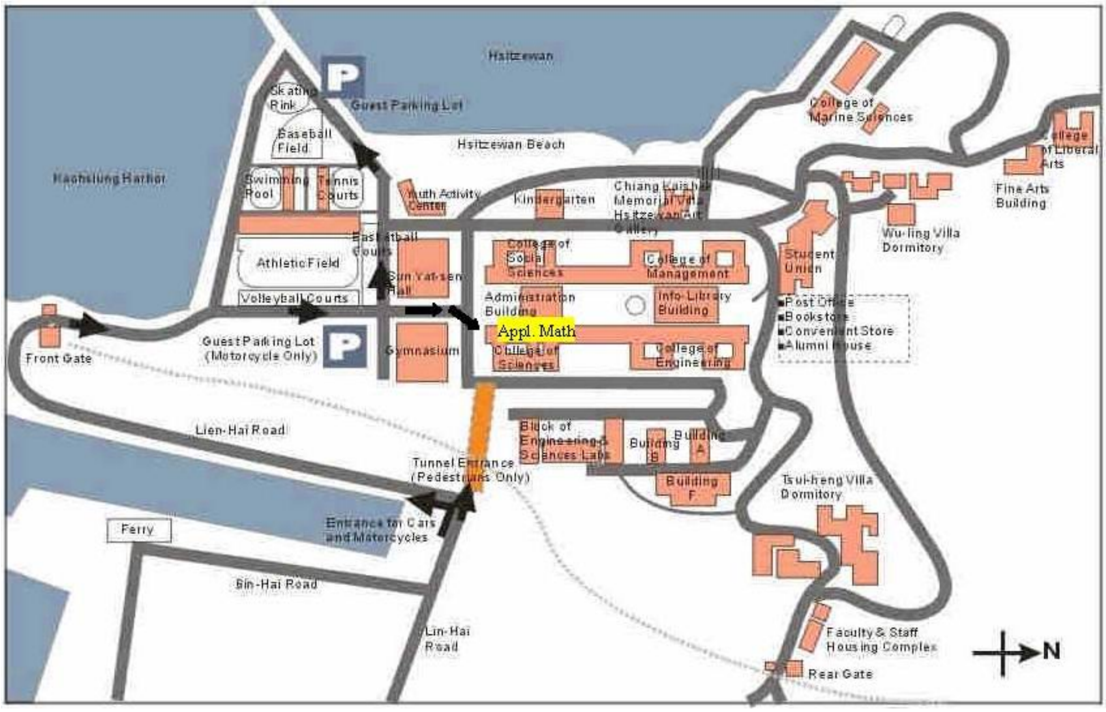
National Taiwan Ocean University (NTOU) was established in 1953 as Provincial Taiwan Maritime Technology College. Eleven years later, in 1964, we became a maritime college that offered bachelor's and master's degrees in various maritime fields. During this period, the college was supported by the Taiwan Provincial Government of the Republic of China. In 1979, we became a national institution, and were renamed the National Taiwan College of Marine Science and Technology. A decade later, in 1989, the college became a full-fledged university.

At present, NTOU has six colleges - Maritime Science and Management, Life Sciences, Ocean Science and Resource, Engineering, Electrical Engineering and Computer Science, as well as Humanities and Social Sciences. These Colleges house a total of 28 undergraduate departments and graduate institutes.

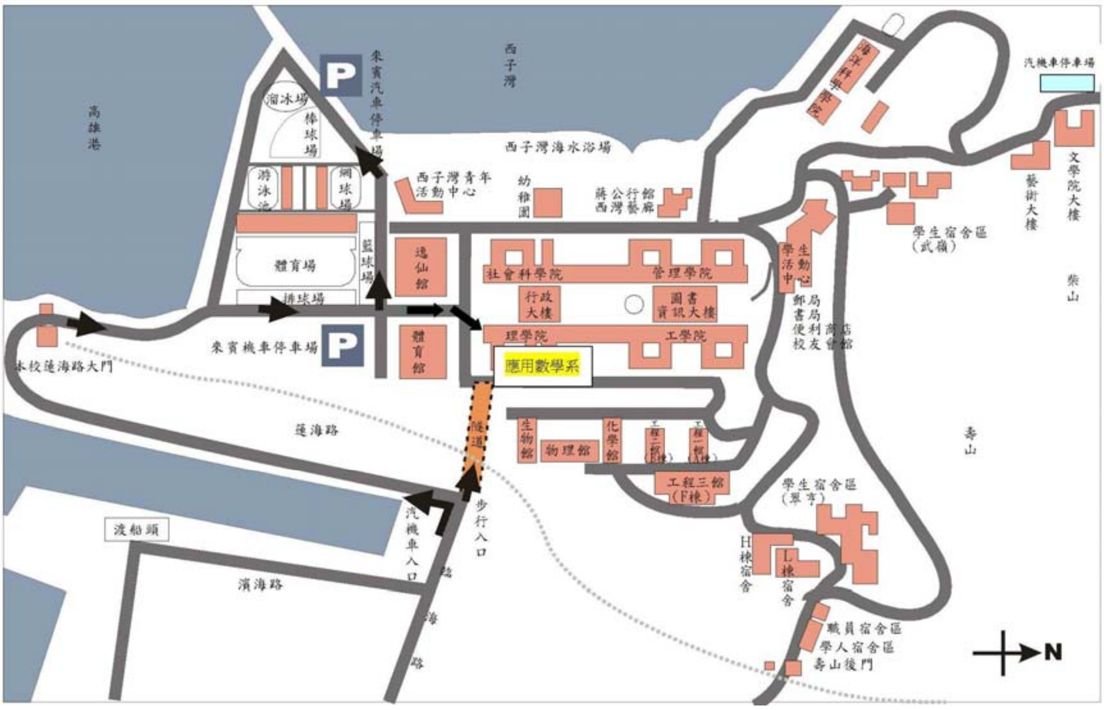
As we look at the progress over the last 52 years, the University has undergone great growth and change and is now recognized as one of the nation's most important centers of learning and scholarship, especially in the marine sciences and maritime studies.

Responding to the changing needs of society and the widening interests of our students, we are developing into a comprehensive university with a unique maritime focus. We aim to be an internationally known institution of higher education. Toward this end, our social sciences and liberal arts programs are developing as a core part of education at NTOU. The University is rapidly moving toward providing a positive learning environment and culture for intellectual and personal growth, with ocean interests as our unique characteristic.

National Sun Yat-sen University Campus Locations

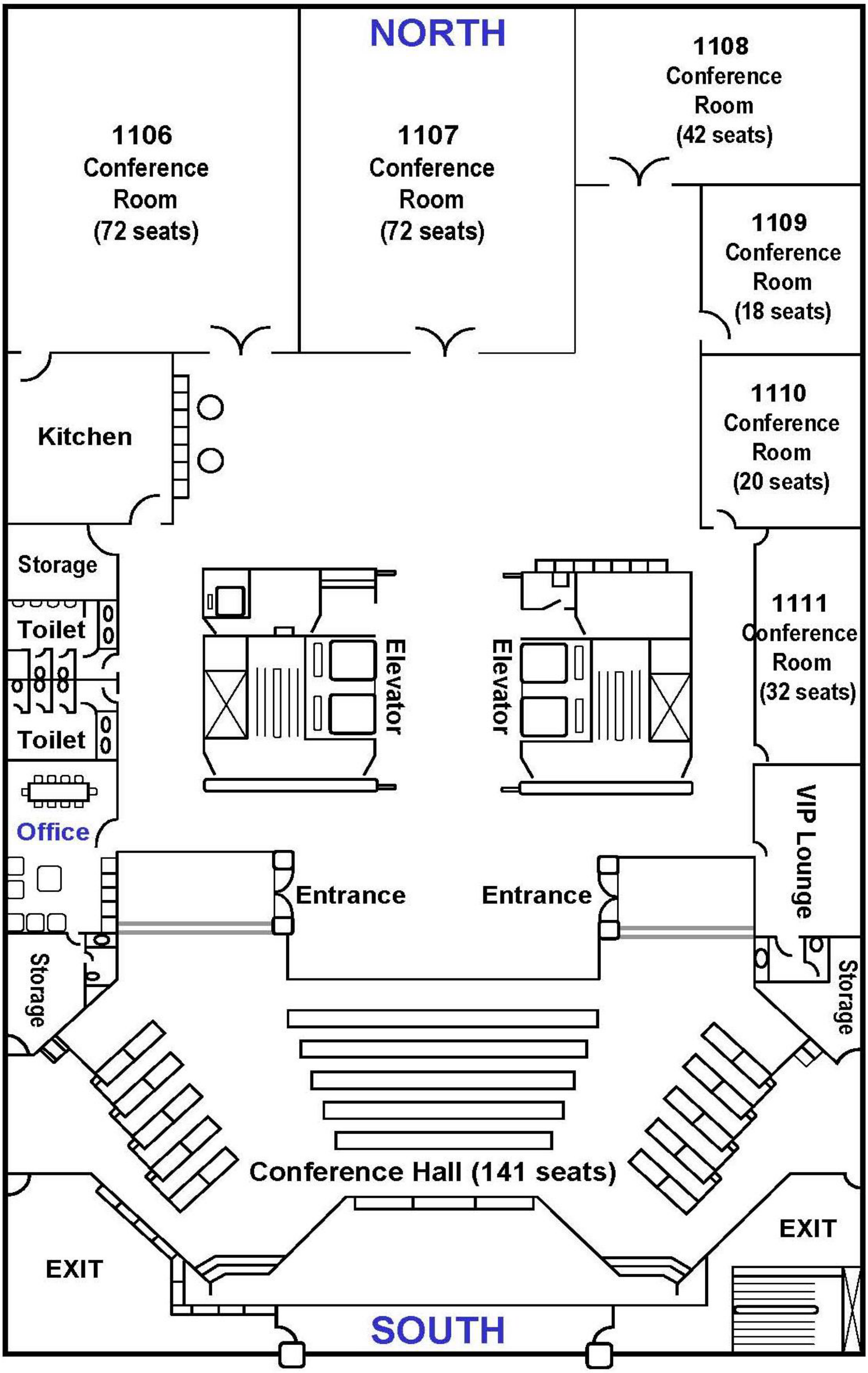


English version



Chinese version

The 11th Floor Plan of Info-Library Building



Program of the Joint Trefftz / MFS Conference 2011

<table>
<tr><th>Time</th><th>March 15 (Tue)</th><th>March 16 (Wed)</th><th>March 17 (Thu)</th><th>March 18 (Fri)</th></tr>
<tr><td>9：00 ~ 9：40</td><td rowspan="2">Registration
Tea / Coffee / Snacks</td><td>Plenary Talk 3</td><td>Plenary Talk 5</td><td>Plenary Talk 6</td></tr>
<tr><td>9：40~10：00</td><td colspan="3">Tea Break</td></tr>
<tr><td>10：00~11：00</td><td>Opening Ceremony
Plenary Talk 1</td><td>Parallel Session 4</td><td>Parallel Session 7</td><td>Parallel Session 9</td></tr>
<tr><td>11：00~11：20</td><td colspan="4">Tea Break</td></tr>
<tr><td>11：20~12：20</td><td>Parallel Session 1</td><td>Parallel Session 5</td><td>Parallel Session 8</td><td>Parallel Session 10</td></tr>
<tr><td>12：20~14：00</td><td colspan="4">Lunch</td></tr>
<tr><td>14：00~14：40</td><td>Plenary Talk 2</td><td>Plenary Talk 4</td><td rowspan="6">Group Photo
City Tour</td><td>Parallel Session 11</td></tr>
<tr><td>14：40~15：00</td><td colspan="2">Tea Break</td><td>Tea Break</td></tr>
<tr><td>15：00~16：00</td><td>Parallel Session 2</td><td>Parallel Session 6</td><td>Parallel Session 12</td></tr>
<tr><td>16：00~16：20</td><td>Tea Break</td><td rowspan="3">Harbor Cruise</td><td>Tea Break</td></tr>
<tr><td>16：20~17：20</td><td>Parallel Session 3</td><td>Parallel Session 13</td></tr>
<tr><td>17：20~18：00</td><td>Music Show</td><td rowspan="2">Shopping</td></tr>
<tr><td>18：00~20：00</td><td>Reception</td><td>Dinner</td><td>Banquet</td></tr>
</table>

★ Parallel sessions include invited and contributed talks. Two sessions hold in the same time. Each talk has 20 minutes.

Program of Plenary Talks

Place: Conference Hall

Plenary Talk 1	**Speaker: I. Herrera** Unified theory of differential operators acting on discontinuous functions and of matrices acting on discontinuous vectors	**Chair: Z. C. Li**
Plenary Talk 2	**Speaker: A. Karageorghis** The MFS for inverse problems	**Chair: C. S. Chen**
Plenary Talk 3	**Speaker: B. Šarler** RBF based solution framework for solving multiphysics and multiscale problems	**Chair: D. L. Young**
Plenary Talk 4	**Speaker: Z. H. Yao** Some knowledgegained from my 30 years investigation on boundary element methods	**Chair: H.-K. Hong**
Plenary Talk 5	**Speaker: R. Schaback** Kernel-based meshless methods for solving PDEs	**Chair: A. H.-D. Cheng**
Plenary Talk 6	**Speaker: A. P. Zielinski** Miscellaneous open problems in the regular boundary collocation approach	**Chair: T. T. Lu**

Brief Program of Invited and Contributed Talks

Place: Conf. Hall	Place: Room1107	Place: Conf. Hall	Place: Room1107	Place: Conf. Hall	Place: Room1107	Place: Conf. Hall	Place: Room1107
Parallel Session 1		**Parallel Session 2**		**Parallel Session 3**		**Parallel Session 4**	
Chair: G. R. Liu	***Chair: Q. H. Qin***	***Chair: S. C. Chiang***	***Chair: C. S. Liu***	***Chair: W. C. Yeih***	***Chair: T. L. Horng***	***Chair: E. Kita***	***Chair: K. Grysa***
G. R. Liu #075	Q. H. Qin #026	S. C. Chiang #088	C. S. Liu #020	W. C. Yeih #043	T. L. Horng #049	E. Kita #019	K. Grysa #015
A. Naji #051	W. S. Shyu #048	V. Kompis #027	B. Van Genechten #005	W. Fujisaki #012	Y. T. Lee #045	Y. W. Chen #030	A. GhannadiAsl #003
T. Zhang #072	N. A. Dumont #047	D. C. Lo #077	C. L. Kuo #024	L. J. Young #036	J. W. Lee #044	C. Y. Ku #078	W. M. Lee #039
Parallel Session 5		**Parallel Session 6**		**Parallel Session 7**		**Parallel Session 8**	
Chair: *H.-K. Hong*	***Chair: Y. M. Wei***	***Chair: A. Tadeu***	***Chair: H. Power***	***Chair: Y. C. Hon***	***Chair: M. Ciałkowski***	***Chair: J. D. Yau***	***Chair: N. Nishimura***
H.-K. Hong #067	Y. M. Wei #082	A. Tadeu #065	H. Power #079	Y. C. Hon #011	J. A. Kołodziej #009	T. S. Jiang #033	N. Nishimura #087
B. Y. Ding #001	T. Shigeta #074	J. António #066	C. J. Xu #086	M. Mierzwiczak #028	M. Ciałkowski #016	J. D. Yau #034	T. Matsumoto #084
X. P. Xie #069	M. G. Lee #037	C. T. Wu #052	C. H. Hsiao #059	C. M. Fan #038	W. N. Zhang #083	C. M. Fan #040	A. GhannadiAsl #002
Parallel Session 9		**Parallel Session 10**		**Parallel Session 11**		**Parallel Session 12**	
Chair: V. M. A. Leitão	***Chair: J. Sladek***	***Chair: W. Chen***	***Chair:*** *C. J. S. Alves*	***Chair: C. C. Tsai***	***Chair:*** *C. S. Huang*	***Chair: C. Gáspár***	***Chair: C. M. Fan***
V. M. A. Leitão #070	J. Sladek #013	A. H.-D. Cheng #061	C. J. S. Alves #089	C. C. Tsai #006	C. S. Huang #004	C. Gáspár #008	C. M. Fan #035
L. Ling #085	C. Y. Lin #054	W. Chen #021	A. Uscilowska #007	M. H. Gu #055	I. L. Chen #042	Y. M. Zhang #014	X. Wei #068
K. H. Chen #041	T. S. Jiang #032	H. Htike #010	C. S. Wu #053			Y. Gu #022	Z. J. Fu #017
Parallel Session 13							
Chair: L. Ling	***Chair: J. A. Kołodziej***						
C. C. Hsiang #063	C. P. Sun #057						
T. F. Chen #062	Y. L. Chan #56						
Y. H. Huang #060	C. H. Chen #058						

Program of Invited and Contributed Talks

Parallel Session 1

Date: 03/15 (Tue) Time: 11:20~12:20 Place: Conf. Hall Chair: *G.R. Liu*

Authors	Title
G. R. Liu	Meshfree methods by weakened weak (W2) formulations
A. Fili A. Naji Y. Duan	Coupling three-field and meshless mixed Galerkin methods using radial basis function to solve parabolic equation
Tie Zhang Zheng Li	Optimal error estimate and superconvergence of the DG method for first-order hyperbolic problems

Date: 03/15 (Tue) Time: 11:20~12:20 Place:Room 1107 Chair: *Q. H. Qin*

Authors	Title
H. Wang Q. H. Qin X. P. Liang	Solving the nonlinear Poisson-type problems with F-Trefftz hybrid finite element model
W. S. Shyu	SH-wave scattering at a semi-cylindrical hill and a semi-cylindrical alluvial basin by hybrid method
N. A. Dumont D. H. Mosqueira	Hybrid finite elements for strain gradient elasticity: theory and patch tests

Parallel Session 2

Date: 03/15 (Tue) Time: 15:00~16:00 Place:Conf. Hall Chair:*S.C. Chiang*

Authors	Title
S. C. Chiang C. J. Tsou	A numerical scheme for a class of singular integro-differential equations with controls
V. Kompiš et al.	Parallel computational models for composites reinforced by short fibers
D. C. Lo C. S. Chen D. L. Young	A new embedding finite element method for viscous incompressible flows with complex immersed boundaries on Cartesian grids

Date: 03/15 (Tue) Time: 15:00~16:00 Place:Room 1107 Chair: *C. S. Liu*

Authors	Title
C. S. Liu	The method of fundamental solutions for solving the backward heat conduction problem with conditioning by a new post-conditioner
B. Van Genechten et al.	An efficient Wave Based Method for solving Helmholtz problems in three-dimensional bounded domains
C. L. Kuo C. S. Liu J. R. Chang	A collocation Trefftz method with a post-conditioner for solving 2D Helmholtz problems in arbitrary domains with high wave numbers

Parallel Session 3

Date: 03/18 (Fri) Time: 16:20~17:20 Place: Conf. Hall Chair: *W. C. Yeih*

Authors	Title
W. C. Yeih et al.	Solving the stress intensity factor for a planar crack by using the modified multiple-source Trefftz method
W. Fujisaki T. Fujisawa T. Teranishi	Consideration on effectiveness of the MFS to linear notch mechanics
L. J. Young	Some numerical applications in fracture of materials

Date: 03/15 (Tue) Time: 16:20~17:20 Place:Room1107 Chair: *T. L. Horng*

Authors	Title
T. L. Horng	Fast Chebyshev pseudospectral Poisson solver for all kinds of boundary conditions via diagonalization
Y. T. Lee J. T. Chen	Anti-plane shear problems containing several elliptical holes and/or inclusions
J. W. Lee J. T. Chen	Resonance and focusing of an elliptical harbor by using the null-field BIEM

Parallel Session 4

Date: 03/16 (Wed) Time: 10:00~11:00 Place: Conf. Hall Chair: *E. Kita*

Authors	Title
R. Fujiwara N. Sekiya E. Kita	Energy derivative valuation using radial basis function
Y. W. Chen et al.	Numerical simulation of the two-dimensional sloshing problem using a multi-scaling Trefftz method
C. Y. Ku	Radial basis function methods incorporated with a manifold-based exponentially convergent algorithm for solving partial differential equations

Date: 03/16 (Wed) Time: 10:00~11:00 Place:Room1107 Chair: *K. Grysa*

Authors	Title
K. Grysa	Indirect Trefftz method in the non-stationary problems
A. GhannadiAsl	Application of indirect Trefftz boundary method in solving the Helmholtz equation in 2D finite domain
W. M. Lee	The collocation Trefftz method for acoustic scattering by multiple elliptical cylinders

Parallel Session 5

Date: 03/16 (Wed) Time: 11:20~12:20 Place: Conf. Hall Chair:*H.-K. Hong*

Authors	Title
H.-K. Hong	Clifford-valued boundary methods for anisotropic vector potential problems
B. Y. Ding et al.	The coupling solutions of the dynamic partial differential equations and decomposition of a generalized function δ
X. P. Xie L. Chen Y. Wu	Hybrid stress finite volume method for linear elasticity problems

Date: 03/16 (Wed) Time: 11:20~12:20 Place: Room1107 Chair: *Y. M. Wei*

Authors	Title
Yi-Min Wei Tzon-Tzer Lu Hung-Tsai Huang Zi-Cai Li	Effective condition number for weighted linear least squares problems and applications to the Trefftz methods
T. Shigeta D. L. Young	Condition number and the related mathematical study on boundary meshless methods for the laplace equation in an exterior unbounded domain
M. G. Lee Z. C. Li P. C. Chu	Corner and crack singularity of different types of boundary conditions for linear elastostatics and their numerical solutions

Parallel Session 6

Date: 03/16 (Wed) Time: 15:00~16:00 Place: Conf. Hall Chair: ***A. Tadeu***

Authors	Title
A. Tadeu I. Castro	Wave propagation involving solid-fluid interaction using a BEM/TBEM and MFS coupling formulation
J. António A. Tadeu	The method of fundamental solutions used to simulate sound wave propagation inside a sound absorbent enclosed space
C. T. Wu D. L. Young	Application of the method of fundamental solutions and the generalized Lagally theorem to the hydrodynamic force on solid body with external singularity

Date: 03/16 (Wed) Time: 15:00~16:00 Place: Room1107 Chair: ***H. Power***

Authors	Title
H. Power C. Nieto M. Giraldo	Boundary element solution of Stokes nano-flow between curved surfaces with linear and nonlinear boundary condition
C. J. Xu	An unstructured nodal spectral-element method for the Navier-Stokes equations
C. H. Hsiao D. L. Young	The singularity method: on the motion of a rotating sphere in unsteady Stokes flows

Parallel Session 7

Date: 03/17 (Thu) Time: 10:00~11:00 Place: Conf. Hall Chair: *Y. C. Hon*

Authors	Title
Y. C. Hon M. Li	A localized direct meshless method for ill-posed inverse problems
M. Mierzwiczak	The inverse determination of the thermal contact resistance between components of unidirectionally reinforced composite
H. F. Chan C. M. Fan	The modified collocation Trefftz method and exponentially convergent scalar homotopy algorithm for the inverse boundary determination problem

Date: 03/17 (Thu) Time: 10:00~11:00 Place:Room1107 Chair: *M. Ciałkowski*

Authors	Title
J. A. Kołodziej M. Mierzwiczak	Application of the method of fundamental solutions for inverse problem of determination of the Biot number
M. Ciałkowski J. A. Kołodziej	Solution of inverse design problem of cooling of annulus by the method of fundamental solutions and minimization of intensity of entropy production
W. N. Zhang	An algorithm for Melnikov functions and applications

Parallel Session 8

Date: 03/17 (Thu) Time: 11:20~12:20 Place: Conf. Hall Chair: *J. D. Yau*

Authors	Title
T. S. Jiang Z. L. Jiang J. Kolibal	A new numerical method for one-dimensional time-dependent Schrodinger equation using radial basis functions
S. R. Kuo J. D. Yau	Applications of Trefftz method to torsionally loaded bucking of a circular plate with a center hole
Y. C. Liu et al.	The least squares Trefftz method with external source for the eigenfrequencies of waveguides

Date: 03/17 (Thu) Time: 11:20~12:20 Place:Room1107 Chair: *N. Nishimura*

Authors	Title
N. Nishimura	Calderon preconditioners for periodic FMMs in wave transmission problems
T. Matsumoto et al.	Shape and topology optimizations using BEM and a level set based method
A. GhannadiAsl	A wavelet-Galerkin boundary element method for the 2D Helmholtz problem

Parallel Session 9

Date: 03/18 (Fri) **Time: 10:00~11:00** **Place:Conf. Hall** **Chair:** ***V.M.A. Leitão***

Authors	**Title**
V. M. A. Leitão	Flexible local approximations using fundamental solutions
L. Ling	Applicability and optimality of the method of fundamental solutions
C. T. Chen K. H. Chen F. L. Jhone	New estimation technique of the optimal source points location in the method of fundamental solutions for multi-connected problem

Date: 03/18 (Fri) **Time: 10:00~11:00** **Place: Room1107** **Chair:** ***J. Sladek***

Authors	**Title**
C. S. Chen J. Sladek	The method of fundamental solutions verse the method of particular solutions
C. Y. Lin M. H. Gu D. L. Young	The localized method of particular solutions for the Burgers' equations via the Cole-Hopf transformation
T. S. Jiang et al.	A new numerical method of particular solutions for one-dimensional time-dependent Schrödinger equations

Parallel Session 10

Date: 03/18 (Fri) Time: 11:20~12:20 Place: Conf. Hall Chair: *W. Chen*

Authors	Title
A. H.-D. Cheng	Multiquadric and its shape parameter
W. Chen Y. Gu	Recent advances on singular boundary method
H. Htike W. Chen J. J. Yang	Material point method with RBF interpolation

Date: 03/18 (Fri) Time: 11:20~12:20 Place:Room1107 Chair: *C. J. S. Alves*

Authors	Title
C. J. S. Alves	The connection between MFS and RBF interpolation
A. Uscilowska D. Berendt	An implementation of the method of fundamental solutions for the dynamics of a plate large displacement
C. S. Wu D. L. Young	Method of fundamental solutions for the vibroacoustic analysis

Parallel Session 11

Date: 03/18 (Fri) Time: 14:00~14:40 Place: Conf. Hall Chair: *C. C. Tsai*

Authors	Title
C. C. Tsai P. H. Lin	On the exponential convergence of method of fundamental solutions
M. H. Gu C. Y. Lin D. L. Young	The Eulerian-Lagrangian method of fundamental solutions for the hyperbolic system problem

Date: 03/18 (Fri) Time: 14:00~14:40 Place:Room1107 Chair: *C. S. Huang*

Authors	Title
C. S. Huang C. S. Chen K.-H. Lin	On the shape parameter of the MFS-MPS scheme
I. L. Chen	Interaction of water waves with vertical cylinder using the method of fundamental solutions

Parallel Session 12

Date: 03/18 (Fri) Time: 15:00~16:00 Place: Conf. Hall Chair: ***C. Gáspár***

Authors	**Title**
C. Gáspár	Regularization techniques for the method of fundamental solutions
Y. M. Zhang W. Z. Qu J. T. Chen	An average source meshless method for solving the potential problems
Y. Gu W. Chen	Investigation on nearly-boundary solutions by singular boundary method

Date: 03/18 (Fri) Time: 15:00~16:00 Place: Room1107 Chair: ***C. M. Fan***

Authors	**Title**
H. H. Li C. M. Fan H. F. Chan	Solving the direct and inverse Stokes problems by the boundary knot method and Laplacian decomposition
X. Wei W. Chen	The boundary knot method for Poisson and inhomogeneous biharmonic problems
Z. J. Fu W. Chen Q. H. Qin	Heat conduction analysis in functionally graded materials by two boundary collocation methods

Parallel Session 13

Date: 03/18 (Fri) Time: 16:20~17:20 Place: Conf. Hall Chair: *L. Ling*

Authors	Title
C. C. Hsiang D. L. Young	2D Shallow Water Equations by Localized Meshless Methods
T. F. Chen et al.	The local radial basis function differential quadrature method for 1D shallow water equations
Y. H. Huang et al.	Local radial basis function-based differential quadrature method for 2-D free surface problem

Date: 03/18 (Fri) Time: 16:20~17:20 Place:Room1107 Chair: *J. A. Kołodziej*

Authors	Title
D. L. Young C. P. Sun	Pricing options for the jump-diffusion models by the local differential quadrature method
Y. L. Chan et al.	The interpolation techniques based on the local radial basis function differential quadrature method
D. L. Young C. H. Chen	A linear iterative investigation for MFS with EEM to solve the 3D nonhomogeneous diffusion equation

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2. Having determined your audience the next step is to decide what you want to tell them. In planning your presentation you must first answer the question "***why do I want to talk to these people*?**"
3. Structure your presentation in a similar way to your written paper. First introduce yourself and the presentation, then move to the main body of the paper. Having done that, draw your conclusions and describe future work.
4. The purpose of a presentation is to make the audience want to understand more about your subject. You should assume that people have not read your paper, so you should try to make them want to read it.
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Abstracts of Plenary Talk

Unified theory of differential operators acting on discontinuous functions and of matrices acting on discontinuous vectors

Ismael Herrera and Ernesto Rubio
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Abstract

For the application of discontinuous functions in the area of numerical methods for partial differential equations (NMPDE), there are mainly two approaches: Trefftz methods and discontinuous Galerkin (dG) methods. The Theory of Differential Equations in Discontinuous Piecewise-Defined-Functions, introduced and developed by Herrera [3], constitutes a unified framework for these procedures. On the other hand, nowadays the application of high performance computing to the solution of PDEs is progressing at a very swift pace. Among the new computational resources parallel computing is outstanding. In turn, the most effective means for applying parallel computing are domain decomposition methods (DDM). Since the 1980s parallel computing has received considerable attention by the NMPDE community and at present it is recognized that non-overlapping DDMs are the most effective. Most of the work done up to recently for this latter kind of methods had been restricted mainly to symmetric and positive definite problems. However, recently Herrera [1-7] has introduced a new formulation in which symmetric and nonsymmetrical problems are handled in a unified manner, thereby producing a systematic non-overlapping and preconditioned DDM for non-symmetric matrices. These procedures are carried out in vector-spaces whose elements are discontinuous, using a unified theory of differential operators acting on discontinuous functions and of matrices acting on discontinuous vectors, to which this plenary lecture is devoted.

***Keywords*: Trefftz method, discontinuous Galerkin, domain decomposition methods, Steklov-Poincaré operator, multipliers-free DDM, Lagrange multipliers

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Kernel--based meshless methods for solving PDEs

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Abstract

This survey starts from general kernels, but soon focuses on the special use of kernels in meshless methods. Kernels provide particular and fundamental solutions, and lead to efficient meshless methods without numerical integration. They allow error bounds and convergence results for unsymmetric methods like the MLPG, and if time permits, the underlying theory will be reviewed, together with a recent extension to nonlinear problems. Finally, certain singularity--free harmonic kernels in 2D and 3D will be presented. They reduce the solution of potential problems to a simple interpolation on the boundary and allow a rigid mathematical analysis.

RBF based solution framework for solving multiphysics and multiscale problems

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Abstract

Structure of a new meshless solution framework for solving multiphysics and multiscale solid and fluid mechanics problems is presented. The physics on the macroscale is based on the continuum mechanics and on the microscale on the cellular automata principles. The solution procedure is defined on a set of nodes which can be non-uniformly distributed. The domain and boundary of interest are divided into overlapping influence areas. On each of them, the fields are represented by the radial basis functions collocation or least squares approximation on a related sub-set of nodes. The transition rules are defined for a set of nodes on the influence area in case of cellular automata modelling. The timestepping is performed in an explicit way. All governing equations are solved in their strong form, *i.e*. no integrations are performed. The polygonisation is not present. The possible change of the shape of the domain is described by activation of additional nodes and by the movement of the boundary nodes through the computational domain, respectively. The solution can be easily and efficiently adapted in node redistribution and/or refinement sense, which is of utmost importance when coping with fields exhibiting sharp gradients Step by step benchmarking of the method is represented, followed by industrial examples such as casting, rolling, and heat treatment. The results of the new approach are compared with the analytical solutions, well documented bench-mark solutions and commercial packages. The method turns out to be extremly simple to code, accurate, inclusion of the complicated physics can easily be looked over. The coding in 2D or 3D is almost identical.

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Some knowledgegained from my 30 years investigation on boundary element methods

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Abstract

The author devotes in the investigation on boundary element method for 30 years. It is fully realized that the boundary element method is an important supplement of the finite element method. The research on BEM should fully display its own advantages, including: as a dimension reduction method, it is advantageous for the description of the actual geometrical shape; by using of the analytical fundamental solution, it has higher accuracy of the analytical-numerical combined methods; it is advantagious to deal with the problems related to infinite domain, and it is also easier to treat very large finite domain; it is easier to find out the singularity of stress, and also suitable to deal with the problem with singular stresses, such as the fracture problems; it is also advantageous to deal with the boundary nonlinear problems, such as the elastic contact problem; the display of the results obtained by BEM should not simply copy that applied in FEM. The conventional BEM is difficult to deal with the large-scale problems because the matrix of the BEM equation system is dense. This draw back has been overcome in recent years by introducing the fast multipole method. But to develop the practical applications on large scale scientific and

engineering problems, it is required to work hard on the development of the software of fast multipole BEM. In addition, it should be mentioned that the selection of the elements is important for the computation using BEM, the simplest element is not always reasonable.

The MFS for inverse problems

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Abstract

The method of fundamental solutions (MFS) is a relatively new technique which can be used for the numerical solution of certain boundary value problems and initial/boundary value problems. The ease with which it can be implemented and its effectiveness have made it very popular for the solution of a large variety of problems arising in science and engineering. Recently, it has been used extensively for a particular class of such problems, namely inverse problems. We attempt to review the applications of the MFS to the various classes of inverse and related problems, over the last few years. Some of the several issues related to the implementation of the MFS to such problems are discussed and some representative numerical results are presented.

Miscellaneous open problems in the regular boundary collocation approach

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Abstract

Certain characteristic aspects of the generalized Trefftz method have been analysed. In this method the trial functions identically fulfill the governing differential equations of the considered Boundary Value Problem (BVP) inside the whole region Ω including its boundary Γ. To approximate well the solution, these trial functional sets (called Trefftz- or T-functions) should be complete. The above requirements are

fulfilled by at least two types of functional systems: the general solutions of homogeneous governing equations of the problem, as well as its fundamental solutions with singularities outside the investigated region Ω. In the latter case, the location of singularities presents an open problem, which is discussed in detail and illustrated by numerical examples. In both above cases the solution inside Ω is obtained as a sum of regular functions with unknown coefficients. The coefficients are determined from the regular boundary integral equations resulting from the boundary and connectivity conditions of the problem.

In large majority of cases the above integrals are calculated numerically. It has be proved, that *the numerical integral fit of the approximate solution is a specific version of collocation in the integral control points with integration weights. Therefore, the notion of boundary collocation can be considered as more general*. In this case the situating of control points and the choice of collocation weights is an open question of the considered problem. It has been discussed in detail.

Usually the boundary Γ consists of segments. From the physical point of view, certain sub-regions of Ω, and thus certain segments, can be more important than the other. Hence, we can influence the accuracy of solution in the important regions by introducing additional weights in the segments of Γ near these regions. The additional weights are also important in connectivity of different regions, because of difference between the essential and natural continuity conditions. This open problem has been discussed and illustrated by numerical examples.

The solution of the BVP defined in a large and complex area Ω often requires division of Ω into subregions. This process is called substructuring. Several typical methods of sub-structuring are presented on numerical examples. Their advantages and drawback are discussed. It should be underlined that they still require careful investigations but, so far, the results are very encouraging.

There are several directions in which the generalized Trefftz method (or the Regular Boundary Collocation approach) can be developed. In opinion of the author, the most promising are structural and multidisciplinary optimization, inverse formulations of mechanics and certain nonlinear problems. The current developments in these directions are presented and discussed.

Abstracts of Invited and Contributed Talks

Parallel Section 1

Meshfree methods by weakened weak (W2) formulations

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Abstract

This paper provides a brief overview on meshfree methods. We introduces first the fundamental theory for general numerical methods, the G space theory that allows the use of much more types of methods/techniques to create shape functions for numerical methods. Weakened weak (W2) formulations can then be used to construct many meshfree methods. We prove that the numerical methods developed based on the W2 formulation will be spatially stable, and convergent to exact solutions. We next present examples of some of the possible W2 models, and show the major properties of these models: 1) it is variationally consistent in a conventional sense, if the solution is sought from a proper H space (compatible cases); 2) it passes the standard patch test when the solution is sought in a G space with discontinuous functions (incompatible cases); 3) the stiffness of the discretized model is reduced compared to the FEM model and even the exact model, allowing us to obtain upper bound solutions with respect to both the FEM and the exact solutions; 4) the W2 models are less sensitive to the quality of the mesh, and triangular meshes can be used without any accuracy problems. These properties and theories have been confirmed numerically via examples.

Keywords: Numerical methods, meshfree methods, FEM, real-time computation, solution bound, inverse analysis

Coupling three-field and meshless mixed Galerkin methods using radial basis function to solve parabolic equation

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Abstract

In this work we extend the method based on the coupled meshless mixed Galerkin approximation using radial basis function with the three-field domain decomposition method to solve parabol equation. Numerical results of transport-diffusion flow in porous media are given.

Optimal error estimate and superconvergence of the DG method for first-order hyperbolic problems

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Abstract

We consider the original discontinuous Galerkin method for the first-order hyperbolic problems in d-dimensional space. We show that, when the method uses polynomials of degree k, the L_2-error estimate is of order $k+1$ provided the triangulation is made of rectangular elements satisfying certain conditions. Further, we show the $O(h^{2k+1})$-order superconvergence for the error on average on some suitably chosen subdomains(including the whole domain) and their outflow faces. Moreover, we also establish a derivative recovery formula for the approximation of the convection directional derivative which is superconvergent with order $k + 1$.

Solving the nonlinear Poisson-type problems with F-Trefftz hybrid finite element model

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Abstract

A hybrid finite element model based on F-Trefftz kernels or the fundamental solution is formulated for the solution of the nonlinear Dirichlet problems associated with two-dimensional nonlinear Poisson-type equations including nonlinear Poisson-Boltzmann equation and diffusion-reaction equation. The nonlinear right-hand forcing term in the nonlinear Poisson-type equation is frozen by introducing the imaginary terms at each Picard iteration step, and then the induced Poisson problem is solved by the present hybrid finite element model involving element boundary integrals only, coupling with the particular solution method with radial basis function interpolation. The numerical accuracy of the present method is investigated by numerical experiments in complex domain with various nonlinear forcing functions, and at the same time, both the iteration residual and the inter-iteration difference are monitored for convergence demonstration.

Keywords: Nonlinear Poisson-type equation, hybrid finite element method, fundamental solution, radial basis function

SH-wave scattering at a semi-cylindrical hill and a semi-cylindrical alluvial basin by hybrid method

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Abstract

The response of surface motion inside and near an irregular area embedded into an elastic half-plane is investigated for the case of incident plane SH wave. The irregular areas include a semi-cylindrical hill and semi-cylindrical alluvial basin. The results of simple geometric shapes such as semi-circular or semi-elliptical canyons are obtained by using separation of variables and expansion of the solution in a basis of orthogonal functions. In this paper, based on a variational formalism which proposed by Mei(1980), a hybrid method which combing the finite element and series expansion method is implemented to solve the scattering problems. We define a substructure which enclosing the irregular area can easily be formulated by finite element method. The unknown boundary data called the scattered waves can be formulated through a series representation with unknown coefficients. Due to the continuity condition at the interface, therefore, the unknown coefficients of this series representation are treated as generalized coordinates and can be easily formulated by the same variational principle. The expansion function of the series representation is constituted of basis function, each basis function is constructed by Lamb's solution and satisfies both traction free condition at ground surface and radiation condition at infinity. The merit of the hybrid method is that the flexibility of finite elements offers the greatest advantage to model the irregular area. We use a simple mapping function to calculate the coordinates of the irregular region. The node numbers of the finite elements and the arrangement of the elements are the same as different areas.

Keywords: semi-cylindrical hill, semi-cylindrical alluvial basin, SH wave, scattering wave, hybrid method, mapping function

Hybrid finite elements for strain gradient elasticity: theory and patch tests

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Abstract

The paper presents an application of the strain gradient theory to two-dimensional and three-dimensional hybrid finite elements. The implemented strain gradient theory, as proposed by Aifantis, is a "weakly" nonlocal elasticity model for homogeneous materials based on one additional parameter (besides the Lamé's constants): the "material characteristic length" g2. It is shown that the hybrid finite element formulation – as proposed by Pian and generalized by Dumont for finite and boundary elements – provides a natural conceptual framework to properly deal with the interelement compatibility of the normal displacement gradients, in which “corner nodes” are not an issue. The theoretical formulation leads to the definition of static boundary conditions interpreted as normal and tangential double forces, besides the boundary forces of the classical elasticity. General families of 2D and 3D finite elements are presented. The consistency of the formulation is assessed by means of generalized patch tests and in terms of the spectral properties of the resultant matrices for linear, quadratic and cubic 2D elements. Moreover, convergence tests are performed for some irregularly-shaped numerical models.

Keywords: Gradient elasticity, variational methods, hybrid finite element, Hellinger-Reissner potential, patch tests, spectral properties

Parallel Section 2

A numerical scheme for a class of singular integro-differential equations with controls

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Abstract

We consider numerical methods for a class of singular integro-differential equations with initial conditions. For this specific type of integro-differential equations, it can be transformed into first order hyperbolic partial differential equations. By applying nonconforming finite element methods on space and keeping time as a variable, we establish a semi-discretized scheme and then use an ordinary differential equation solver for this semi-discretized scheme. We report the numerical solutions of a class of singular integro-differential equations with controls provided different initial conditions.

Parallel Computational Models for Composites Reinforced by Short Fibers

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Abstract

The aspect ratio of the short fibres reinforcing composite material is often 103:1-106:1, or even more. Method of Continuous Source Functions (MCSF) developed by authors using 1D continuous source functions distributed along the fibre axis enables to simulate the interaction of fibre with the matrix and also with other fibres. 1D source functions serve as Trefftz (T-) functions, which satisfy the governing equations inside the domain (matrix) and boundary conditions on the fibre-matrix boundaries are satisfied in collocation points in the least square (LS) sense along the fibre boundaries. The source functions are defined by Non-Uniform Rational

B-Splines (NURBS). Because of large gradients, large number of collocation points and many NURBS shape functions are necessary to simulate the interaction. Moreover, the matrices solving the problem numerically are full. In our model, only the interactions of each two fibres is solved by elimination and then the complex interaction of all fibres in a patch of fibres and the matrix is completed by iteration steps in order to increase efficiency of computations. For heat problem, material of fibres is supposed to be super-conductive in the first steps. The energy balance condition in each fibre enables to find temperature change of each fibre by the interaction with the other fibre in the first iteration steps. The next iteration steps enable to correct the temperature changes of the fibres by complex interaction of all fibres and the matrix and distribution of the source functions inside the fibres are obtained. Temperatures and heat flows in the control volume enable then to define homogenized material properties for corresponding patch of the composite material.

Keywords: Computational mechanics, composite materials, short fibers, 1D continuous Trefftz functions, LS collocation

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A new embedding finite element method for viscous incompressible flows with complex immersed boundaries on cartesian grids

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Abstract

We develop an innovative numerical method for solving the two-dimensional incompressible viscous Navier-Stokes equations in complex physical domains. Our method is based on an underlying non-uniform Cartesian grid and finite element discretizations of the Navier-Stokes equations in velocity-pressure variables form. Geometry representing stationary solid obstacles in the flow domain is embedded in the Cartesian grid and special discretizations near the embedded boundary ensure the accuracy of the solution in the cut cells. In order to comprehend the complexities of the viscous flows with immersed boundaries, we adopt a new compact interpolation scheme near the immersed boundaries that allows us to retain second-order accuracy and conservation property of the solver. The interpolation scheme is designed in the spirit of the shape function of finite element. In order to verify the proposed methodology, a first step attempted is the simulation of the low and moderately high Reynolds number of flow past a circular cylinder. The results obtained of the drag and left coefficients show excellent agreement with the results available in the literature. Then, uniform flow around a pair of staggered cylinders is investigated. We simulate time-based variation of the flow phenomena for uniform flow past a pair of cylinders with various streamwise and transverse gaps between two cylinders. Additionally, the detail flow characteristics, such as velocity distribution, pressure and vorticity fields are sketched in this study. Thus, it is convinced that the combined an interpolation scheme and FE discretizations are robustness and accuracy of solving 2D fluid flow with complex immersed boundaries.

Keywords: finite element, interpolation scheme, immersed boundaries, drag and left coefficients.

The Method of Fundamental Solutions for Solving the Backward Heat Conduction Problem with Conditioning by a New Post-Conditioner

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Abstract

We consider a backward heat conduction problem (BHCP) in a slab, by subjecting to noisy data at a final time. The BHCP is known to be highly ill-posed. In order to stably and numerically solve the BHCP, we employ a new post-conditioner to reduce the condition number of the resulting linear system obtained from the method of fundamental solutions. The present method can retrieve very well the initial data with a high order accuracy. Several numerical examples of the BHCP demonstrate that the present method is applicable, even for those of strongly ill-posed ones with a large value of final time.

Keywords: inverse problem, backward heat conduction problem, strongly ill-posed problem, post-conditioner.

An efficient Wave Based Method for solving Helmholtz problems in three-dimensional bounded domains

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Abstract

This paper discusses the use of a Wave Based prediction method for the analysis of time-harmonic interior acoustic problems. Conventional element-based prediction methods, such as the finite element method, are most commonly used, but they are restricted to low-frequency applications. The Wave Based Method is an alternative deterministic technique which is based on the indirect Trefftz approach. It is computationally very efficient, allowing the analysis of problems at higher frequencies. Numerical validation examples show the enhanced computational efficiency of the Wave Based Method as compared to conventional element-based methods.

A collocation Trefftz method with a post-conditioner for solving 2D Helmholtz problems in arbitrary domains with high wave numbers

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Abstract

In this study, a collocation Trefftz method is proposed to form a boundary type meshless method for solving the Helmholtz problems in arbitrary plane domains. The solution is expressed by a linear combination of the T-complete functions and higher order terms are needed for higher wave number problem. The resultant linear systems obtained by the conventional collocation Trefftz method are ill-posed due to the oscillations of the higher order terms even for solving direct problems, and it is difficult to compute the strength of the bases accurately. To overcome this deficiency we propose a new method to reduce the condition number of the resultant matrix by using a novel post-conditioner, which is obtained by the multiple length scales collocation

Trefftz method for solving the Laplace equation. Several numerical examples are provided to demonstrate the ability that the post-conditioner can reduce the condition number effectively. The present results are accurate and stable since the ill-posed systems are transformed into well-posed ones successfully by the presently proposed post-conditioner.

Keywords: Collocation Trefftz method, post-conditioner, Helmholtz problems, high wave number.

Parallel Section 3

Solving the stress intensity factor for a planar crack by using the modified multiple-source Trefftz method

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Abstract

In this paper, the stress intensity factor for a planar crack is calculated by the modified multiple-source Trefftz method. To simulate the discontinuous behaviors of physical quantities across crack surface, we locate a Trefftz source point at the crack tip and construct necessary basis functions to simulate the discontinuous behaviors along a semi-infinitely crack surface. For a finite length crack with two crack tips, we then need to arrange two source points at two crack tips and construct corresponding basis functions. In order to reduce the ill-posed behavior resulting from the conventional Trefftz method, we adopt the modified Trefftz method which uses the concept of characteristic length. It is found that the current proposed method can easily treat planar crack problems which are difficult for the conventional Trefftz method. Four numerical examples are demonstrated to show the validity of current approach. Numerical results show the current approach is promising and highly accurate.

Consideration on effectiveness of the MFS to linear notch mechanics

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Abstract

It is well known that the method of fundamental solutions (MFS) based on the collocation method has simple program architecture, but the stability of accuracy depends on each problem in case of high stress concentration factor (SCF). One of the authors has been improving the source loads of the MFS program [1, 2] to calculate the stress at the notch tip correctly. We compared the accuracy by using conventional point loads and the accuracy by the equally dispersed point loads (EDPL). It is found that the improved MFS gives the good accuracy even if SCF>5 under some conditions. The index on the balance of force was also proposed and was found to be effective to confirm the accuracy of the maximum stress. On the view of product design, the maximum elastic stress in a notched body is not a unique parameter that controls the failure of the product. Therefore, we should use the proper measures of severity in a notched body in order to predict the strength of real objects from the strength of specimens. This idea is called linear notch mechanics which was proposed by Nisitani. To utilize linear notch mechanics to evaluate the failure behavior, we should calculate two parameters correctly. One is the maximum stress at the notch tip and the other one is the gradient of the stress near the notch tip. When the measures of severity are the same in a specimen and a real object under the condition of small-scale yielding, the same elastic-plastic fields are assured and the same phenomena occur in both. In this study, we give some examples which shows that the MFS is better than FEM to calculate those two parameters correctly.

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Some numerical applications in fracture of materials

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Abstract

In linear elastic fracture mechanics the stress and strain magnification at the crack tip is characterized by stress intensity factors which have been obtained for a large variety of loading and specimen geometries. However, in reality, the stress of course, will not go to infinity at the crack tip if the plastic zone be considered. In this research some realistic specimens which considered the plastic zone will be discussed by using the boundary element method (BEM). The BEM offers important advantages over "domain" type solutions such as finite element method (FEM). One of the most interesting features of the method is the much smaller system of equations and considerable reduction in the data required to run a problem. In addition, since conditions at infinity are incorporated in the boundary integral directly the BEM is also well suited to problems solving with infinite domains such as the center crack problem in infinite media for which classical domain methods are unsuitable. The results will show the potential application of the BEM.

Fast Chebyshev pseudospectral Poisson solver for all kinds of boundary conditions via diagonalization

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Abstract

Poisson equation for a simple 2D or 3D rectangular domain is frequently encountered in scientific and engineering problems, and a fast Poisson solver would be needed. Poisson equation using Chebyshev pseudospectral method is usually solved in two ways. One is discretizing the Laplace operator to a large matrix based on tensor product of collocation derivative matrix and then solved by sparse solvers. Since the formed matrix is very large even for moderate resolution, it is very time consuming to solve and suffers insufficient computer memory frequently. The other is using iterative method which is also time consuming when resolution is high and sometimes has poor convergence when pre-conditioning is not applied. Actually, an ultra-fast Poisson solver via diagonalization using eigenvectors of collocation derivative matrix has been developed as early as 1987. However, it only works with Dirichlet boundary conditions exclusively, and no Neumann or Robin boundary condition is allowed. This pitfall has severely restricted its usage. The current work removed this serious constraint by adopting penalty method to manipulate boundary conditions of all kinds, and it computes basically as fast as the original one with Dirichlet boundary condition only. The spectral accuracy is obtained when resolution is enough to resolve solution structure and a penalty weight is selected appropriately. This penalty method can be also extended to solve Helmholtz equation and cylindrical Poisson equation.

Keywords: Poisson equation, Chebyshev pseudospectral method, collocation, derivative matrix, diagonalization.

Anti-plane shear problems containing several elliptical holes and/or inclusions

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Abstract

In this paper, we focus on an infinite plane containing several elliptic holes and/or inclusions subject to remote shear by using the null-field integral equation instead of the complex potential method. Here, not only elliptic holes but also elliptic inclusions buried in an infinite plane are considered. For fully employing the elliptic geometry, fundamental solutions were expanded into the degenerate form by using an addition theorem in terms of the elliptic coordinates. Boundary densities are also described by using the eigenfunction expansion. It is worthy of noting that the Jacobian terms exist in the degenerate kernel, boundary density and contour integral; however, these Jacobian terms would cancel out to each other and the orthogonal property is preserved in the process of contour integral. Finally, two examples of an infinite plane containing one and two elliptical inclusions are given to verify the validity of the present approach. A special case of a crack is also addressed.

Keywords: antiplane elasticity, Laplace problem, elliptical inclusion, null-field integral equation

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Resonance and focusing of an elliptical harbor by using the null-field BIEM and hybrid method

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Abstract

In this paper, the water-wave resonance of an elliptical harbor is studied by using the semi-analytical approach. The method is based on the null-field boundary integral equation method (BIEM) in conjunction with degenerate kernels and the eigenfunction expansions. The problem is decomposed into two regions by employing the concept of taking free body. One is an elliptical harbor, and the other is a problem of half-open sea with a coastline subject to the impermeable (Neumann) boundary condition. It is interesting to find that the SH wave impinging on the hill can be formulated to the same mathematical model. After finding the analogy between the harbor resonance and hill scattering, focusing of the water wave inside an elliptical harbor is also examined. Several numerical examples of elliptical harbor problems with different opening entrances are used to verify the validity of the present formulation. Finally, numerical results are compared well with those of the hybrid method.

Keywords: harbor resonance, focusing, null-field, BIEM degenerate kernel, hybrid method

Parallel Section 4

Energy derivative valuation using radial basis function

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Abstract

A derivative contract is a financial contract whose value is derived from the values of one or more underlying assets, reference rates, or indices of asset values or reference rates. In this study, we will focus on the energy derivative. The energy derivative is a derivative contract in which the underlying asset is based on energy products including oil, natural gas and electricity, which trade either on an exchange or over-the-counter.

When the derivative price V depends on the energy spot price S and the time t, the price V should satisfy the differential equation:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+\alpha(\mu-\lambda-\ln S)S\frac{\partial V}{\partial S}-rv=0 \qquad (1)$$

The expiration condition of the European future option price is given as follows

$$V(F,T)=\max(F(T)-E,0) \qquad \text{(for Call)}$$
$$V(F,T)=\max(E-F(T),0) \qquad \text{(for Put)}$$

where the parameter E is the strike price on the expiration date T. The function F is given as

$$F(t,T)=\exp\left[e^{-\alpha(T-t)}\ln S+\left(1-E^{-\alpha(T-t)}\right)\left(\mu-\lambda-\frac{\sigma^2}{2\alpha}\right)+\frac{\sigma^2}{4\alpha}\left(1-e^{-2\alpha(T-t)}\right)\right] \qquad (2)$$

Use of equation (2) gives the following equation from equation (1)

$$\frac{\partial V}{\partial t}+\frac{e^{-2\alpha(T-t)}}{2}\sigma^2F^2\frac{\partial^2 V}{\partial F^2}-rV=0 \qquad (3)$$

The above equation is discretized with Crank-Nicolson method and the radial bases function approximation $V\approx\sum\lambda_j^t\phi_j$. Finally, we have

$$HV^{t+\Delta t}=GV^t \qquad (4)$$

The symbols H and G are differential operators.

Equation (4) is solved backward from the expiration date T to the date of purchase in order to valuate the derivative price.

Keywords: energy derivative, radial basis function, meshless method

Numerical simulation of the two-dimensional sloshing problem using a multi-scaling Trefftz method

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Abstract

Here, we develop a multi-scaling Trefftz method (MSTM) for the Laplace equation associated with the group-preserving scheme (GPS) to describe nonlinear sloshing behavior of the incompressible, non-viscous, and irrotational fluid. Chen et al. [29] proposed that the characteristic length of the Trefftz method and the concept of controlled volume could be used to overcome numerical errors and dissipation in the simulation of the sloshing problem. However, the nonlinear dependence of the characteristic length on initial conditions was neglected in the numerical development. In addition, this study presents a numerical method with automatically adaptive computational steps for describing the nonlinear sloshing behavior as well as for satisfying the conservation of mass at each time step. A comparison of the results of the present study with those in the literature shows that the numerical results are better than those obtained using previously reported methods. The method developed here presents a simple and stable way to cope with the nonlinear sloshing problem.

Keywords: Laplace equation; Multi-scaling Trefftz method; Mixed-boundary value problem; Sloshing.

Radial basis function methods incorporated with a manifold-based exponentially convergent algorithm for solving partial differential equations

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Abstract

In this study, a Manifold-Based Exponentially Convergent Algorithm (MBECA) combined with radial basis function (RBF) methods for solving partial differential equations (PDEs) is proposed. RBF methods have shown great advantages of the grid free property of the methods, and they are becoming a viable choice as a method for the numerical solution of PDEs. We first employ RBF methods to discretize PDEs into nonlinear algebraic equations. Then, the MBECA is adopted to solve the nonlinear algebraic equations. The MBECA is based on the construction of a space-time manifold to transform a vector function into a time-dependent scalar function by introducing a fictitious time variable. Taking the advantages of the time-dependent scalar function, the proposed novel MBECA does not need to calculate the inverse of the Jacobian matrix and has a great advantage that it is exponentially convergent. Illustration examples demonstrate that the MBECA can dramatically improve the convergence and has a greater performance than other nonlinear equation solvers. Several numerical examples including elliptic type PDEs are examined and compared to exact solutions. Results obtained show that the proposed MBECA combined with radial basis function (RBF) methods can significantly improve the accuracy as well as the convergence for solving PDEs.

Keywords: Manifold-Based exponentially Convergent Algorithm (MBECA), Radial Basis Function (RBF), Jacobian matrix, Partial Differential Equations (PDEs), Non-Linear Algebraic Equations.

Indirect Trefftz method in the non-stationary problems

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Abstract

Trefftz method has been known since the work of Trefftz in 1926. Thereafter the method has been extensively developed in both formulation and application. For a long time the Trefftz method was applied only to stationary problems or to problems in which authors used the method to get rid of time. In late nineties Trefftz method was broadened for the non-stationary problems.

Here the indirect Trefftz method is considered. In such approach, an approximate solution of the initial-boundary problem is a linear combination of Trefftz functions. The unknown coefficients are determined from the conditions of approximate fulfilling the boundary and initial conditions. In the case of inverse problems certain additional conditions are included (e.g. measured or anticipated values of searched solution inside the considered body).

In order to find the approximate solution of a direct or inverse problem frequently the FEM is applied Such approach leads to the so called FEMT (finite element method with Trefftz functions), for stationary problems widely described in the papers of Jiroušek, Herrera, and others. In the case of inverse problems their ill-posedness requires additional terms in an objective functional, type of regularization or penalty terms.

Here some remarks concerning the Trefftz indirect method referred to the non-stationary inverse problems are presented. Certain results for non-stationary problems of heat conduction (among others boundary temperature identification and thermal diffusivity estimation), for beam vibration, for elastokinetics and for wave equation (direct and inverse problem of membrane vibrations) are presented. Three kinds of FEMT are tested for direct and inverse non-stationary problems: (a) FEMT with the condition of continuity of temperature in the common nodes of elements, (b) no temperature continuity at any point between elements and (c) nodeless FEMT (substructuring).

An idea of physical regularization of noisy input data, based on Trefftz function, is discussed. In the problem of membrane vibration the approximate solutions of both, direct and inverse problem, even for slightly disturbed input data ((N(0, 0.01)) are completely wrong. However, an error of approximate solution obtained with the noisy

data smoothed with the use of Trefftz functions practically does not exceed the inaccuracy of the input data.

Trefftz functions for linear partial differential equations are useful also for problems with a heat source moving on a boundary.

Keywords: indirect Trefftz method, non-stationary inverse problems, FEMT, input data smoothing, heat conduction, elastokinetics, vibrations

Application of indirect Trefftz boundary method in solving the Helmholtz equation in 2D finite domain

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Abstract

Trefftz method could be used at the boundary-type solution procedure using regular T-complete functions satisfying the governing equation. Until now, it has been mainly applied to numerical analyses of the problems governed with the homogeneous differential equations such as the two and three dimensional Laplace problems and the two dimensional elastic problem without body forces. One considerable advantage of the Trefftz method in comparison with the conventional BEM is that the integral equation is regular and no singular integral is required. In the Trefftz-type boundary solution method, smooth non-singular Trefftz complete solutions are used in formulations, therefore the results will generally be more accurate because of the ease in integrating the integral correctly. This paper describes the application of the indirect Trefftz method to the solution of the Helmholtz equation. The Helmholtz equation is frequently encountered in various fields of engineering and physics. The vibration of membranes and the water wave diffraction problems in offshore structural engineering are the two examples. Therefore, indirect formulation using complete and non-singular systems of Trefftz functions for the Helmholtz equation are posed in this paper. In the indirect formulation, the solution is approximated by superposition of the regular T-complete functions with unknown coefficients. Then, the unknown parameters are determined so that the approximate solution satisfies boundary conditions. The present scheme applies to some examples in order to examine the numerical properties. The numerical results indicate that a nonsingular complete series solution can be obtained by the present method and the results show that the aforementioned method is effective for solving of Helmhlotz

equation.

Keywords: indirect Trefftz method, boundary-type solution, T-complete functions and Helmholtz equation

The collocation Trefftz method for acoustic scattering by multiple elliptical cylinders

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Abstract

In this paper, the collocation Trefftz method is presented to solve the problem of the acoustic scattering of a plane sound wave by multiple elliptical cylinders. To satisfy the Helmholtz equation in the elliptical coordinate system, the scattered field is formulated in terms of angular and radial Mathieu functions. The boundary conditions are satisfied by uniformly collocating points on the boundaries. The acoustic pressure at each boundary point is directly calculated in each elliptical coordinate system. In different coordinate systems, the normal derivative of the acoustic pressure is calculated by using the appropriate directional derivative, an alternative to the addition theorem. By truncating the multipole expansion, a finite linear algebraic system is derived and then the scattered field can be determined according to the given incident acoustic wave. Once the total field is calculated as the sum of the incident field and the scattered field, the near field acoustic pressure along scatterers and the far field scattering pattern can be determined. The proposed results of scattered fields by two, three and four elliptical-cylindrical scatterers are compared with those of available analytical solutions and the BEM to verify the validity of the present method. Finally, the effects of the space among scatterers and the incident wave number and angle on the scattered fields are investigated in this paper.

Keywords: Collocation Trefftz method, Acoustic scattering, Helmholtz equation, Mathieu functions, Elliptical cylinder, Far field scattering pattern.

Parallel Section 5

Clifford-valued boundary methods for anisotropic vector potential problems

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Abstract

The coefficients of most anisotropic vector potential problems such as anisotropic elasticity, piezoelasticity, etc. possess the properties of major and first minor symmetries. The symmetries are taken advantage of to decompose twice the anisotropic vector potential operator into the Laplacian operator spectra, and each Laplacian operator is then factored into the Dirac (or Cauchy-Riemann) operators. The harmonic and monogenic (or holomor-phic) fundamental solutions (FS) and particular solutions (PS) are synthesized in such a way reversing the earlier twice spectral decompositions and Clifford factorization as to constitute the FS and PS of the n-dimensional anisotropic vector potential equation. Using the FS, we derive Clifford-valued singular boundary integral equations (BIE) for bounded domains with corners, and propose boundary methods based upon the FS, PS, and BIE.

The coupling solutions of the dynamic partial differential equations and decomposition of a generalized function δ

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Abstract

If the solutions of a dynamic partial differential equation are couple, there are lots of troubles to deal with it. A process on decouple of fast and slow dilational waves to solve the Biot's two-phase dynamic partial differential equation with decomposition the δ function was been shown in this paper. The results of solving equation are compared with previous solutions. The situation of good agreement of both is not only to express that the results obtained in this paper are reasonable, and indicate that a performing of decomposition δ function may be effective and convenient to solve some partial differential equation.

Keywords: solution, coupling, dynamic partial differential, decomposition, generalized function δ

Hybrid stress finite volume method for linear elasticity problems

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Abstract

A hybrid stress finite volume method is proposed for linear elasticity equations. In the approach, a finite volume formulation is used for the equilibrium equation, and a hybrid stress quadrilateral finite element discretization is used for the constitutive equation with continuous piecewise isoparametric bilinear displacement interpolation and two types of stress approximation modes. The method is shown to be free from Poisson-locking, in the sense that the error bounds in the a priori error estimates are independent of Lame constant λ. Numerical experiments confirm the theoretical results.

Effective condition number for weighted linear least squares problems and applications to the Trefftz methods

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Abstract

In this talk, we extend the effective condition number for weighted linear least squares problem with both full rank and rank-deficient cases. First we apply the effective condition number for the linear algebraic equations of the notorious Hilbert

matrix, $\mathbf{H} \in \mathbf{R}^{n\times n}$. The traditional condition number is huge for not small *n*, but the effective condition number is small. When $n = 10$, $\text{Cond} = 0.16\times 10^{14}$. On the other hand, for 10^8 number of random right hand vectors, the maximal effective condition number is less than 10^5. Furthermore, we apply the effective condition number to the collocation Trefftz method (CTM) [1] for Laplace's equation with a crack singularity, to prove that $\text{Cond\_eff} = O(\sqrt{L})$ and $\text{Cond} = O(L^{\frac{1}{2}}(\sqrt{2})^L)$, where *L* is the number of singular particular solutions used. The Cond grows exponentially as *L* increases, but Cond_eff is only $O(\sqrt{L})$. The small effective condition number explains well the high accuracy of the TM solution, but the huge Cond can not.

Keywords: Condition number, effective condition number, perturbation, weighted linear least squares problem, collocation Trefftz method, singularity problem.

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Condition number and the related mathematical study on boundary meshless methods for the Laplace equation in an exterior unbounded domain

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Abstract

The method of fundamental solution (MFS) can easily and efficiently solve a boundary value problem of the Laplace equation in an exterior unbounded domain. The numerical solution obtained by the MFS is accurate although the corresponding matrix equation is ill-conditioned. The reduction of the condition number of the matrix is however required to stabilize the numerical solution. The MFS proposed in this study uses proper basis functions that strictly satisfy a condition at infinity, and the corresponding matrix is approximately obtained and has a lower condition number by way of the modified Trefftz method (MTM). The mathematical expressions of the corresponding condition numbers are derived. The solvability of the MTM and our

method is mathematically proven. Thereby, the optimal parameter minimizing the condition number can explicitly be given in a mathematical form. Numerical experiments show that the condition number is reduced and that the numerical solution by the proposed method is more accurate than the one by the conventional method.

Keywords: condition number, exterior unbounded domain, Laplace equation, method of fundamental solution, modified Trefftz method

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Corner and crack singularity of different types of boundary conditions for linear elastostatics and their numerical solutions

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Abstract

The singular solutions at corners and the fundamental solution are essential in both theory and computation. Our recent efforts are made to seek the particular solutions of corner and crack singularity of linear elastostatics, to design new models of corner singularity, and to find their numerical solutions. In [1, 2], a systematic analysis for singularity properties and particular solutions of linear elastostatics is explored, and the singular solutions for corners with the displacement or the free traction boundary conditions have been found. This talk is a continued study of [1, 2, 3], to explore new particular solutions for the case that the displacement and the free traction boundary conditions are subjected to the same corner edge. Explicit particular solutions have been found for any angle $\Theta \in (0, 2\pi]$; this is different from [1, 2, 3] where the explicit solutions only with $\Theta = \pi$ and $\Theta = 2\pi$ can be obtained. In this talk new singularity models with L-shaped domain and other non-rectangular domains are designed, and the highly accurate solutions are computed. Moreover, the singularity solutions as $O(r^{\frac{1}{4}})$ and even $O(r^{\frac{1}{7}})$ are found (ref. [1, 2, 3]). To our best knowledge, this is the first time to provide the particular solutions with different boundary conditions on the same corner edge in linear elastostatics. The new particular solutions, new singularity, analysis, and computation in this paper are

important for both theory and computation of linear elastostatics.

Keywords: Singular solutions, fundamental solutions, particular solutions, corner singularity, crack singularity, linear elastostatics, intensity factors, collocation Trefftz method, Trefftz method.

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Parallel Section 6

Wave propagation involving solid-fluid interaction using a BEM/TBEM and MFS coupling formulation

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Abstract

This paper proposes a coupling formulation between the boundary element method (BEM) / the traction boundary element method (TBEM) and the method of fundamental solutions (MFS) for the transient analysis of elastic wave propagation involving solid-fluid interaction. The proposed formulation overcomes the specific limitations of each of these methods. The full domain of the original problem is divided into sub-domains, which are handled separately by the BEM/TBEM or the MFS. The coupling is enforced by imposing the required boundary conditions. The accuracy, efficiency and stability of the proposed algorithms, which use different combinations of BEM/TBEM and MFS, are verified by comparing the solutions against reference solutions. The potential of the proposed procedures is illustrated by simulating the propagation of elastic waves in the vicinity of a fluid-filled borehole, placed in a cracked elastic medium.

Keywords: wave propagation, TBEM/MFS coupling, BEM/MFS coupling, elastic inclusions, cracks, solid-fluid interaction.

The method of fundamental solutions used to simulate sound wave propagation inside a sound absorbent enclosed space

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Abstract

The method of fundamental solutions (MFS) is formulated in the frequency domain to model the wave field generated by a pressure source in an enclosed space (a room). In this formulation the solution is approximated by a linear combination of fundamental solutions generated by virtual sources placed outside the domain in order to avoid singularities The fundamental solutions applied in the present model can simulate acoustic domains such as fluid layers or fluid wedges and are established by using the image source approach. The use of these solutions avoids the placement of collocation points on the horizontal and inclined surfaces and the MFS is only used to couple those acoustic domains along vertical surfaces. Sound absorption can be ascribed to some surfaces by imposing an impedance boundary condition. Results in the time domain are obtained by applying an inverse Fourier transform assuming a source to be modeled as a Ricker wavelet.

Keywords: Method of Fundamental Solutions, Image source technique, Sound wave propagation.

Application of the method of fundamental solutions and the generalized Lagally theorem to the hydrodynamic force on solid body with external singularity

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Abstract

The hydrodynamic force on a solid body in an inviscid, irrotational and incompressible flow with external singularity is calculated by the generalized Lagally theorem and the method of fundamental solutions (MFS). To generate the flow field when there are multiple solid bodies or external singularities present, image singularities, which can be source, vortex, dipole or multipole, located in each body are used to ensure the no-penetration boundary condition. However, the precise positions and strength of these image singularities are in general unknown for irregular shaped body and infinite numbers of image singularities are generated in each body due to the reflection between bodies. These difficulties can be overcome numerically with the application of the MFS which limits the number of the image singularities and provides the positions and strength of these singularities. With these information, the generalized Lagally theorem can describe the hydrodynamic force on each body. The presented method is first validated by calculating the added mass coefficients of a circular cylinder. Then the force on a circular cylinder with the presence of a source or a vortex is calculated and compared to the analytical solutions. The results are all in agreement with the results in the literature.

Keywords: Lagally theorem, Taylor theorem, potential flows, method of fundamental solutions

Boundary element solution of Stokes Nano-flow between curved surfaces with linear and nonlinear boundary condition

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Abstract

The numerical simulation of Micro/Nano fluid flow through the solution of governing equations based on continuum models has to be done under the consideration of appropriate slip boundary conditions to account for the velocity jump at the solid-fluid interface. The liner slip boundary condition states a relation between the tangential shear rate and the fluid-wall velocity differences and has been successfully used in reproducing the characteristics of many types of flows (e.g. slit flows, rotating curved mixers, microbearings, among others), where the shear rate at solid-fluid interfaces remains linear because of the geometry smoothness. Despite this, there are some situations for which this linear dependency fails leading to unrealistic behaviour since the slip length becomes nonlinear and singular like is the case of spreading liquids and corner flows, where a variation in the slip length is suitable due to the presence of regions of higher shear stress and shear rate. The previous behaviour was summarized in an expression for the slip condition at a solid-liquid interface establishing the variation in the slip length in terms of the square root of the tangential shear rate as well. Moreover incorrect flow behaviours has been previously reported in some numerical works dealing with flow fields confinement by curved surfaces, due to wrong evaluation of the local tangential shear rate at the wall surfaces.

In this work a boundary integral equation formulation for Stokes slip flow, based on the normal and tangential projection of the Green's integral representational formulae for the Stokes velocity field, which directly incorporates into the integral equations the local tangential shear rate at the wall surfaces, is presented. The resulting boundary integral equation for the slip flow is function of the tangential projection of the shear rate, allowing the evaluation of the nonlinear terms that appear

due to the inclusion of the universal slip flow boundary condition. The Boundary Element Method (BEM) is employed to solve the resulting projections of the integral equations coupled with the slip condition as function of surface traction in the tangential direction. The equation system is evaluated iteratively turning the nonlinear term into a nonhomogeneous constant vector by using results from previous iteration.

The proposed BEM formulation is used to simulate flow between curved rotating geometries: i.e. concentric and eccentric Couette and single rotor mixers. The numerical results obtained for the concentric Couette mixer are validated with the corresponding analytical solutions under linear and nonlinear boundary condition, showing excellent agreements. Results obtained in this work extend the use of BEM for the study of microfluid flow, allowing the developed of more complex Micro/Nano fluidic applications.

With the idea of comparing the proposed approach with more simpler alternative scheme, the case of concentric Couette flow with linear and nonlinear boundary condition was also solved with the Method of Fundamental Solutions (MFS) showing that for simple geometric this approach is very efficient.

An unstructured nodal spectral-element method for the Navier-Stokes equations

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Abstract

In this talk, we present an unstructured nodal spectral-element method for the Navier-Stokes equations. The method is based on a triangular and tetrahedral rational approximation and an easy-to-implement nodal basis which fully enjoys the tensorial product property. It allows arbitrary triangular and tetrahedral mesh, affording greater flexibility in handling complex domains while maintaining all essential features of the usual spectral-element method. The existence and uniqueness of the approximation to the Stokes equations are proved, together with an error estimation for the velocity. The details of the implementation and some numerical examples are provided to validate the efficiency and flexibility of the proposed method.

The singularity method: on the motion of a rotating sphere in unsteady Stokes flows

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Abstract

The singularities of potential flow such as the sources, sinks, vortices and doublets have been well known for more than one hundred years. For steady Stokes flows, the singular solution also had been derived since the work of Lorentz (1897). For the following development of the singularity method, a family of fundamental solutions called Stokeslet, Rotlet, Stokeslet doublet, Stresslet, and so on was investigated, which had been further employed to construct exact solutions to exterior and interior steady Stokes flow problems, but non-stationary Stokes flow problems are seldom presented. In this paper, the fundamental solution of an unsteady Rotlet (also called a couplet) for the unsteady Stokes equation is derived. The surface force and torque exerted on a fluid sphere centered at the pole of the singularity is also presented. The couple on a sphere starting to rotate with an arbitrary differentiable angular velocity $\boldsymbol{\omega}_{\mathbf{p}}(t)$ in low Reynolds number flows is calculated by applying the fundamental solution of an unsteady Rotlet. The results are compared with exact solutions by Feuillebois and Lasek (1977). Because the feature of the singularity method for steady Stokes flow is that the force and torque are given by the total strength of the singularities, it is also calculated and compared for unsteady Stokes flow.

Keywords: singularity method, unsteady Stokes flow, Rotlet

Parallel Section 7

A localized direct meshless method for ill-posed inverse problems

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Abstract

In this talk we present a localized direct meshless computational method using approximate particular solutions for solving some ill-posed inverse problems of Poisson equation. The Method of Approximate Particular Solutions (MAPS) has recently been developed and demonstrated to be effective in solving various types of partial differential equations (PDEs). The globally implemented MAPS, however, involves the solution of a dense matrix, which is highly ill-conditioned, and hence impractical for handling real large-scale science and engineering problems. We develop a localized scheme for MAPS, which gives a well-conditioned and sparse matrix, and demonstrate its potential to solve real large-scale problems. Furthermore, we successfully combine the localized MAPS with the Method of Fundamental Solutions (MFS) to solve some Cauchy problems, which are typical ill-posed inverse problems. Numerical examples in two-dimensional space are given to illustrate the effectiveness and stability of the proposed method.

The inverse determination of the thermal contact resistance between components of unidirectionally reinforced composite

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Abstract

In this work a boundary collocation Trefftz method has been presented for two-dimensional steady-state heat conduction analysis of fiber reinforced composites. The heat flow in a composite is a complex phenomenon and can only be understood through a proper micromechanical analysis. The solution of the heat conduction problem is usually based on the solution of heat transfer equation at microstructure

level in repeated element of an array [1]. The calculations are carried out for a composite with the fibers arranged in a matrix in a regular manner by square grid (Fig. 1a) with imperfect thermal contact between the constituents. The heat conduction problem of composites with imperfect thermal contact was considered in papers [2,3]. The thermal conductivities of constituents (fibers λ_f and matrix λ_m, $F=\lambda_f/\lambda_m$) and the volume fraction of fibers $\varphi=\pi E^2/4$, where $E=a/b$, are known.

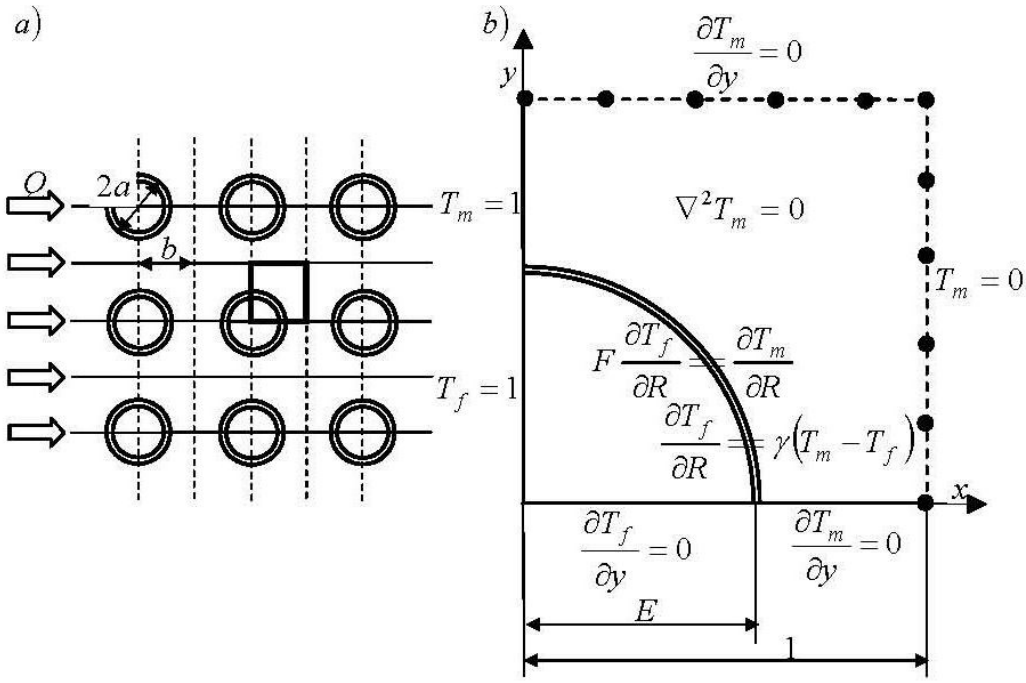

Figure 1 A unidirectional reinforced fibrous composite with fiber arrangement according to a square array for the imperfect thermal contact between fiber and matrix:
a) general view, b) formulation boundary value problem in a repeated element

The purpose of this paper is to propose an analytic-numerical algorithm for determination of the thermal contact resistance γ in unidirectionally reinforced composite. Such a problem can be treated as a kind of the inverse heat conduction problem in inhomogenous material. Because the algorithm for the inverse problem is in some sense based on the solution of the direct problem (Fig.1b) we begin with the boundary Trefftz method to determine the temperature field:

$$T_f = 1 + \sum_{k=1}^{N} w_k R^{(2k-1)} \cos[(2k-1)\theta] \tag{1}$$

$$T_m = 1 + \sum_{k=1}^{N} \frac{w_k}{2} \left[\begin{array}{l} \left(F + 1 + \frac{(2k-1)}{\gamma E}\right) R^{(2k-1)} + \\ -\left(F - 1 - \frac{(2k-1)}{\gamma E}\right) \frac{E^{2(2k-1)}}{R^{(2k-1)}} \end{array} \right] \cos[(2k-1)\theta] \tag{2}$$

and the effective thermal conductivity [1-3]:

$$\frac{\lambda_z}{\lambda_m} = \sum_{k=1}^{N} \frac{w_k}{2} (-1)^k \left[\left(F + 1 + \frac{(2k-1)}{\gamma E} \right) + \left(F - 1 - \frac{(2k-1)}{\gamma E} \right) E^{2(2k-1)} \right] \tag{3}$$

where T_f and T_m are non-dimensional temperatures in the matrix and in the fiber respectively, w_k are unknown coefficients determined from the boundary conditions. For the boundary conditions:

$$T_m = 0 \quad \text{for} \quad x = 1 \tag{4a}$$

$$\frac{\partial T_m}{\partial y} = 0 \quad \text{for} \quad y = 1 \tag{4b}$$

and for the additional condition determined by:

a known value of temperature in N3 points located on the matrix:

$$T_m(R,\theta) = T_i \,.\, i = 1,\ldots,N3 \quad \text{for} \quad R > E\,. \tag{5a}$$

a known value of temperature in $N3$ points located on the upper edge:

$$T_m(R,\theta) = T_i \,.\, i = 1,\ldots,N3 \quad \text{for} \quad Y = 1 \tag{5b}$$

a known value of the effective thermal conductivity (3):

$$\frac{\lambda_z}{\lambda_m}(F,\gamma,E) = \lambda_{eff\,.} \tag{5c}$$

the resistance number and the unknown coefficients w_k, k=1,…, N are calculated.

For the substitution $w_{N+1} = \gamma$ from the collocation [4] of the boundary condition (4a) in N1 points on the right edge and the boundary condition (4b) in N2 points on upper edge of the considered region and from condition (5) we obtain a system of Ne=N1+ N2+ N3 nonlinear equations G with Nu= N+1 with the unknowns w_k which is solved in least square sense:

$$\delta(w_1,\ldots,w_{N+1}) = \sum_{i=1}^{NR} \left[G_i(w_1,\ldots,w_{N+1}) \right]^2 \tag{6}$$

$$\frac{\partial \delta}{\partial w_k} = 2\sum_{i=1}^{NR} \left[G_i(w_1,\ldots,w_{N+1}) \right] \frac{\partial G_i}{\partial w_k} = 0, \qquad k = 1,\ldots,N+1$$

For solving this problem the Levenberg-Marquadt iteration method is used:

$$\sum_{l=1}^{N+1} A'_{k,l} \cdot \delta w_l = b_k, \quad k = 1,2,..,N+1 \tag{7}$$

where: $b_k = -\dfrac{\partial \delta}{\partial w_k}$, $A_{k,l} = \dfrac{\partial^2 \delta}{\partial w_k \partial w_l}$, $A'_{k,l} = \begin{cases} A_{k,l} \cdot (1+\lambda) & k = l \\ A_{k,l} & k \neq l \end{cases}$ is used.

The study compared three different cases of the additional conditions (5). In addition to determining the resistance number γ, the impact of the number of collocation points ($N1$, $N2$) on the quality of the results was examined. Because an iterative method was used the convergence had to be considered. The method was not always quickly convergent.

Keywords: Trefftz Method, Levenberg-Marquadt Method, Thermal Contact Resistance, Unidirectionally Reinforced Composite

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The modified collocation trefftz method and exponentially convergent scalar homotopy algorithm for the inverse boundary determination problem

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Abstract

In this paper, the modified collocation Trefftz method (MCTM) and the exponentially convergent scalar homotopy algorithm (ECSHA) are adopted to analyze the inverse boundary determination problem governed by the biharmonic equation. The position for part of boundary with given boundary condition is unknown and the

position for the rest of boundary with additionally specified boundary conditions is given. Since the spatial position for portion of boundary is not given a priori, it is extremely difficult to solve the boundary determination problem by any numerical scheme. In order to stably solve the boundary detection problem, the MCTM, one kind of boundary-type meshless methods, will be adopted in this study, since it can avoid the generation of mesh grid and numerical integration. In the boundary determination problem governed by the biharmonic equation, the numerical solution of MCTM is expressed as linear combination of the T-complete function. When this problem is considered by MCTM, a system of nonlinear algebraic equations will be formed and solved by ECSHA which will converge exponentially. The unknown coefficients in MCTM and the position of the unknown boundary can be found simultaneously by the evolutionary process of ECSHA. Some numerical examples will be provided to demonstrate the ability and accuracy of the proposed scheme. Besides, the stability of the proposed meshless method will be proven by adding some noise into the boundary conditions.

Keywords: modified collocation Trefftz method, exponentially convergent scalar homotopy algorithm, boundary determination problem, biharmonic equation, boundary-type meshless method

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Application of the method of fundamental solutions for inverse problem of determination of the Biot number

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Abstract

Several functions and parameters can be estimated from the inverse heat conduction problem: static and moving heating sources, material properties, initial conditions, boundary conditions, etc.. This study is limited to the estimation of the heat transfer coefficient for steady state heat conduction problem. This problem is one of the versions of so called Robin inverse problem. Several numerical methods have been proposed for solving the Robin inverse problem, in particular in the context of corrosion detection [5, 8]. Among these methods the most popular one is the boundary element method [1, 3, 7, 9, 10]. Other methods used are Galerkin method [4, 5], finite element method [8, 11], control volume method [2]. The authors usually use the Cauchy data on some part of boundary [2-8] as an additional condition. In the paper

[1] the author uses the inner temperature for the identification of the heat transfer coefficient that is a function of the boundary coordinates.

In this paper the identification of the Biot number is carried out based on the boundary data and knowledge of temperature in some points inside the domain. The method of fundamental solution (MFS) is proposed to solve 2-D inverse problem of determination of the heat transfer coefficient (dimensionless Biot numbers).As of right now, the MFS was applied in inverse heat conduction problems involving the identification of heat sources, boundary heat flux, Cauchy problem, or boundary determination problem. To the best knowledge of the authors, this paper is a first application of this method to the inverse heat conduction concerned with the identification of the Biot number.

The paper deals with the iterative inverse determination of the Biot numbers for a 2-D steady-state heat conduction problem. The MFS is used to solve the 2-D dimensionless heat conduction problem

$$\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}=0\text{, for }\Omega=[0,1]\times[0,1], \qquad (1)$$

with boundary conditions:

$$\theta(X_b,Y_b)=\theta_b,\quad \text{on } \partial\Omega_1,$$
$$\frac{\partial\theta(X_b,Y_b)}{\partial n}=q_b,\quad \text{on } \partial\Omega_2, \qquad (2)$$
$$\frac{\partial\theta(X_b,Y_b)}{\partial n}=-Bi\cdot\theta(X_b,Y_b),\quad \text{on } \partial\Omega_3,$$

where θ is the temperature field, θ_b is dimensionless temperature on the boundary $\partial\Omega_1$, q_b is dimensionless heat flux on the boundary $\partial\Omega_2$, $Bi=\frac{\alpha l}{\lambda}$ is unknown Biot number.

Using the MFS the solution of equation (1) has a form:

$$\theta(X,Y)=\sum_{j=1}^{M}C_j\ln\left(r_j^2\right) \qquad (3)$$

The identification of the unknown value of coefficients Cj of the solution (3) and the identification the value of the Biot number is obtained from collocation of the boundary conditions (2) and additionally from collocation of data from the knowledge of temperature inside the domain:

$$\theta(X_i,Y_i)=\theta_i,\ \ i=1,2,...,N, \qquad (4)$$

For substitution $C_{M+1}=Bi$ the non-linear system of equation r obtained from collocation of the boundary (2) and the additional collocation conditions (3) can be solved in least square sense:

$$E(C_1,\ldots,C_{M+1}) = \sum_{i=1}^{NG} [r_i(C_1,\ldots,C_{M+1})]^2$$

$$\frac{\partial E}{\partial C_k} = 2\sum_{i=1}^{NG} [r_i(C_1,\ldots,C_{M+1})] \frac{\partial r_i}{\partial C_k} = 0, \qquad k = 1,2,..,M+1 \tag{5}$$

using the Levenberg-Marquadt iteration method:

$$\sum_{l=1}^{M+1} A'(k,l)\cdot \delta C(l) = b(k), \quad k = 1,2,..,M+1 \tag{6}$$

where $b(k) = -\frac{\partial E}{\partial C_k}$, $A(k,l) = \frac{\partial^2 E}{\partial C_k \partial C_l}$, $A'(k,l) = \begin{cases} A(k,l)\cdot(1+\lambda) & k = l \\ A(k,l) & k \neq l \end{cases}$.

The accuracy of the proposed method is tested for several different examples in which the the Biot number is described as a constant $Bi = const.$, as a function of coordinate of boundary $Bi = \sum_{j=1}^{J} Bi_j X^{j-1}$ or as a function of temperature $Bi = \sum_{k=1}^{K} Bi_k T^{k-1}$.

The influence of measurement errors on the Biot number and on the identification of temperature field were also investigated l.

Keywords: method of fundamental solutions, inverse problem, Levenberg-Marquadt method, the Biot number

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Solution of inverse design problem of cooling of annulus by the method of fundamental solutions and minimization of intensity of entropy production

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Abstract

In this paper, the application of the method of fundamental solutions to the Cauchy problem in two-connected plane region for steady heat conduction equation is investigated. The method is tested in annular region for which exact solution is known. The influence of the disturbances of data and the distances of source contour from boundary contour is presented in the series of graphs. By numerical experiments it is found that solution considered inverse problem could be obtained without regularization for moderate disturbances of data.

Keywords: inverse problem, Cauchy problem, method of fundamental solutions, two-dimensional heat conduction

An algorithm for Melnikov functions and applications

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Abstract

In this work we study a dynamical system with a complicated nonlinearity, which describes oscillation of a turbine rotor, and give an algorithm to compute Melnikov functions for analysis of its chaotic behavior. We first derive the rotor model whose nonlinear term brings difficulties to investigating the distribution and qualitative properties of its equilibria. This nonlinear model provides a typical example of a system for which the homoclinic and heteroclinic orbits cannot be analytically determined. In order to apply Melnikov's method to make clear the underlying conditions for chaotic motion, we present a generic algorithm that provides a systematic procedure to compute Melnikov functions numerically. Substantial analysis is done so that the numerical approximation precision at each phase of the computation can be guaranteed. Using the algorithm developed in this paper, it is straightforward to obtain a sufficient condition for chaotic motion under damping and periodic external excitation, whenever the rotor parameters are given.

Parallel Section 8

A new numerical method for one-dimensional time-dependent Schrödinger equation using radial basis functions

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Abstract

This paper proposes a new numerical method to solve the one-dimensional time-dependent Schrödinger equations based on the _nite difference scheme by means of multiquadrics (MQ) and inverse multiquadrics (IMQ) radial basis functions. The numerical examples are given to confirm the good accuracy of the proposed methods.

Keywords: One-dimensional Schrodinger equation; Finite difference; Radial basis functions.

Applications of Trefftz method to torsionally loaded bucking of a circular plate with a center hole

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Abstract

This paper presents a theoretical model to investigate torsionally loaded buckling of a circular plate with a center hole using the Trefftz method. The circular plate is regarded as a thin plate and a set of torsional loads are applied along the inner ad outer edges of the plate, where the torsional loads are in self-equilibrium. In order to study the buckling behaviors of such a circular plate, theoretical formulation of the circular plate subject to outer torsional moment will be first developed and then the governing equation and general solution of the torsionally loaded circular plate are derived in an analytical way. With the use of Trefftz method, Eigen-value problem of buckling analysis will be performed to determine the torsional buckling loads. Numerical investigations of a torsionally loaded circular plate with a center hole will be demonstrated to see the torsional buckling behaviors of the circular plate by using the Trefftz method.

Keywords: buckling, circular plate, the Trefftz method, torsional load

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The least squares Trefftz method with external source for the eigenfrequencies of waveguides

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Abstract

In this paper, the least squares Trefftz method (LSTM) with external sources is proposed to analyze the eigenfrequencies of waveguide. The LSTM, the combination of the conventional Trefftz method and the least squares method, can stabilize the numerical scheme and obtain highly accurate numerical solutions for Helmholtz problem. In LSTM, the numerical solution is expressed as the linear combination of T-complete functions and the number of equations is larger than the number of knowns to avoid the ill-conditioning problem in conventional Trefftz method. The determination of the eigenfrequencies of waveguide, governed by the two-dimensional Helmholtz equation, will form an eigenproblem. Using external sources, the eigenproblem will be converted to an inhomogeneous Helmholtz problem. Then the proposed LSTM will be adopted to analyze the inhomogeneous problem. By recording the responses from a series of tests, the eigenfrequencies of waveguides can be found. In comparing with the technique of singular value decomposition and the method of determinant search, the computational cost of the proposed boundary-type meshless scheme for dealing with the eigenproblems can be evidently reduced. Several numerical examples will be provided to validate the proposed LSTM. Both of the transverse magnetic wave and the transverse electric wave will be examined by the proposed meshless method to demonstrate the capability and robustness of the present method.

Keywords: least squares Trefftz method, external source, eigenfrequencies, waveguide, boundary-type meshless scheme

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Calderon preconditioners for periodic FMMs in wave transmission problems

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Abstract

Wave problems in periodic domains have many interesting applications such as photonic crystals and metamaterials in Maxwell's equations and phononic crystals in elastodynamics, etc. Fast multipole methods are effective as solvers of such problems, particularly when the problems are of scattering type. In view of this, we have developed periodic FMMs for various wave problems. We are now interested in preconditioners suitable for periodic FMMs. In this talk we shall discuss our recent efforts on the use of preconditioners based on Calderon's formulae in periodic transmission problems in Helmholtz equation and elastodynamics. In Helmholtz, we shall show that the matrix of the discretised integral equations itself serves as an effective preconditioner if the integral equation is discretised with collocation. This fact leads to a very simple preconditioning scheme with GMRES as the solver. We shall also see that a simple preconditioning is possible with Galerkin discretisation. These preconditioners are shown to be effective near anomalies related to the periodicity of the problem. Finally, we shall discuss similar preconditioners in elastodynamics and have some comments on Maxwell's equations.

Shape and topology optimizations using BEM and a level set based method

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Abstract

The boundary element method (BEM) is applied for shape and topology optimization problems in three-dimensional elastostatics and acoustic problems. Although domain discretization is not needed in the BEM analyses for linear problems, and those with infinite domains, the required storage and computation costs have been very high. However, these drawbacks are now be overcome thanks to the development of the fast computation algorithm based on multipole expansions of the fundamental solutions of the governing differential equation. Hence, the shape optimization problems are now one of the competitive areas of BEM applications. The present study treats a topology optimization method for three-dimensional elastostatics and acoustic problems using the boundary element method and level set-based structural boundary expressions incorporating a fictitious interface energy model. A procedure to generate the boundary elements from the level set function at each iterative step of optimization process is also proposed. Thus, re-meshing of the body in the optimization process and specifying the boundary conditions on the boundaries under their shape modification are now possible. The approach is demonstrated through several numerical examples of optimum designs for compliance of elastic solids and sound scatters in acoustic fields using conventional and fast-multipole boundary element methods.

Keywords: boundary elment method, topology optimization, level-set function, elastostatics, acoustic problem.

A wavelet-Galerkin boundary element method for the 2D Helmholtz problem

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Abstract

The Galerkin method is one of the best known methods for finding numerical solutions to partial differential equations. The Galerkin approach consists of selecting a function basis for the solution space of the equation, projecting the solution onto this basis, and then minimizing the least-squares residual between the exact solution and the projection onto the subspace spanned by the basis functions. In recent years, the use of wavelets has become increasingly popular in the development of numerical schemes for the solution of partial differential equations. In wavelet applications to the PDE problem, one of the most frequently used wavelets are the Daubechies wavelets which have compact support and are an orthonormal basis for square integrable functions. In this paper, the Wavelet-Galerkin method is used with the Daubechies wavelets as basis functions to solve the two dimensional Helmholtz equation. The method converts the differential equation into a system of algebraic equations. Matlab programs have been developed to solve the equations of the Wavelet-Galerkin method with the Daubechies wavelet basis functions. The results show that the Wavelet-Galerkin method with a Daubechies wavelet basis gives efficient and accurate algorithms for the numerical solution of the two dimensional Helmholtz problem. In order to test the accuracy of our numerical solutions from the Wavelet-Galerkin method we have applied the method to a Helmholtz problem with a known exact analytical solution.

Keywords: wavelet-Galerkin method, Daubechies wavelet and 2D Helmholtz equation

Parallel Section 9

Flexible local approximations using fundamental solutions

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Abstract

Traditional finite difference schemes rely on a structured grid of points in order to define the difference operator. To overcome the need for a structured grid, some non-standard approaches have been proposed, which attempt to generalize the distribution of points by relaxing the grid requirements, thus allowing for difference operators to be obtained on non-structured distribution of points. One of those non-standard approaches, the difference schemes with flexible local approximations proposed in (Tsukerman, I 2006), resorts to Trefftz-type functions to construct finite difference operators that are free from any form of structured grid. The same essential characteristic of the Trefftz functions, which is that of being actual solutions of the homogeneous equation for a given problem, is shared by the fundamental solutions, the building blocks of the boundary element method and of the method of fundamental solutions, traditionally used to build boundary-only global approximations in the domain of interest. Given the shared features of both Trefftz functions and fundamental solutions it seems appropriate to build local approximations, in the way proposed by Tsukerman, whereby the Trefftz functions are replaced by fundamental solutions. The use of fundamental solutions in a local framework guarantees accuracy while simultaneously eliminating (or strongly reducing) potential problems due to numerical ill-conditioning that are normally present in boundary-only solution methods. Applications to two-dimensional potential problems (both Laplace and Poisson) are considered. Comparison with other results available in the literature shows that the method is accurate, reliable and may be considered to be an alternative to other numerical methods. The generalization of the grids allows for changing the grid in a quite effective manner: adding patches of grid points in any shape or size or density on different domains is now simplified. This is a good indication on the capabilities of the method for dealing with complex geometries especially when coupled with domain decomposition techniques.

Tsukerman I. A class of difference schemes with flexible local approximation. Journal of Computational Physics 2006; 211:659–699

Applicability and optimality of the method of fundamental solutions

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Abstract

The condition number of a matrix is commonly used for investigating the stability of solutions to linear algebraic systems. Recent meshless techniques for solving partial differential equations have been known to give rise to ill-conditioned matrices, yet are still able to produce results that are close to machine accuracy. In the first half of this talk, we consider the method of fundamental solutions (MFS), which is known to solve, with extremely high accuracy, certain partial differential equations, namely those for which a fundamental solution is known. The effective condition number (ECN) is a sensitivity measure for a linear system; it differs from the traditional condition-number in the sense that the ECN is also right-hand side vector dependent. We revealed the close connection between the ECN and the accuracy of the Method of Fundamental Solutions (MFS) for each given problem. In the second half, we show how the ECN can help achieve the problem-dependent quasi-optimal settings for MFS calculations-that is, determining the position and density of the source points. A series of examples on Dirichlet and mixed boundary conditions shows the reliability of the proposed scheme; whenever the MFS fails, the corresponding value of the ECN strongly indicates to the user to switch to other numerical methods.

New estimation technique of the optimal source points location in the method of fundamental solutions for multi-connected problem

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Abstract

In this presentation, a new estimation technique will be proposed to estimate the numerical error of the method of fundamental solutions (MFS) for solving the multi-connected problem. The successful experiences of simply-connected problem are extended to deal with multiply-connected problems. The sources are distributed on the inner and outer fictitious boundaries, respectively, in multiply-connected domain. The optimal sources location and the optimal number of sources are found without comparison with a given analytical solution of an interested problem. The convergent numerical solutions of the MFS adopting the optimal parameter can be obtained in unavailable analytical solution condition. Finally, several numerical experiments of multi-connected problem are given to demonstrate the validity and accuracy of the error estimation technique.

Keywords: error estimation, optimal parameter, method of fundamental solutions, Trefftz set, discretizing error, simply-connected problem, multi-connected problem, error magnitude, numerical error, convergence rate

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The method of fundamental solutions verse the method of particular solutions

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Abstract

The method of fundamental solutions (MFS) is highly effective for solving homogenous partial differential equations. The MFS loses its attractiveness for solving inhomogeneous equations. In the past, radial basis functions have been used to interpolate the inhomogeneous term and thus obtain a particular solution. Through the available of the particular solutions, one can reduce the inhomogeneous equation to homogeneous equation which allows the MFS to be implemented to obtain homogeneous solution. This is a two stage approach. The alternative approach is to use the same concept of the MFS to solve the inhomogeneous solution directly using the radial basis functions. Instead of using fundamental solution as a basis function for homogeneous equation, we use special type of radial basis functions as a basis function to approximate the inhomogeneous equation directly. The numerical process of the later one is simple and effective. We will make numerical comparison of these two approaches in term of accuracy and simplicity.

The localized method of particular solutions for the Burgers' equations via the Cole-Hopf transformation

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Abstract

The localized method of particular solutions (LMPS) is a meshless numerical method based on the method of particular solutions. The LMPS introduceS the localization to palliate ill-conditions and increase efficiency by transforming the system into sparse matrix, while without sacrificing much accuracy. In this paper the high-order finite difference (FD) scheme is used to discretize the time domain, combining the LMPS as the proposed numerical scheme. The proposed meshless numerical model approximates the solution of Burgers' equations dependent on the Cole-Hopf transformation. The Cole-Hopf formula transforms the Burgers' equations into the diffusion equation for solving the initial value problem. Hence the non-linear system of Burgers' equations can be solved by proposed meshless numerical methods. The numerical results will demonstrate the consistent behaviors of the LMPS in solving the Burgers' equations via the Cole-Hopf transformation. The proposed numerical scheme shows good efficiency, high accuracy and robustness, further application into engineering problem can be expected.

Keywords: Meshless numerical method, localization, method of particular solutions, Burgers' equation, the Cole-Hopf transformation.

A new numerical method of particular solutions for one-dimensional time-dependent Schrödinger equations

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Abstract

Based on the _nite di_erence scheme in time, the method of particular solutions using the radial basis functions is proposed to solve one-dimensional time-dependent Schrödinger equations. Two numerical examples with good accuracy are given to validate the proposed method.

Keywords: One-dimensional Schrödinger equation, Finite di_erence, Particular solutions, Radial basis functions

Parallel Section 11

Multiquadric and its shape parameter

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Abstract

Hardy's multiquadric and its related interpolators have been found to be highly efficient for interpolating continuous, multivariate functions, as well as for the solution of partial differential equations. Particularly, the interpolation error can be dramatically reduced by varying the shape parameter to make the interpolator optimally flat. This improvement of accuracy is accomplished without reducing the fill distance of collocation points, that is, without the increase of computational cost. There exist a number of mathematical theories investigating the multiquadric family of radial basis functions. These theories are often not fully tested due to the computation difficulty associated with the ill-conditioning of the interpolation matrix. This paper overcomes this difficulty by utilizing arbitrary precision arithmetic in the computation. The issues investigated include conditional positive definiteness, error estimate, optimal shape parameter, traditional and effective condition numbers, round-off error, derivatives of interpolator, and the edge effect of interpolation.

Recent Advances on Singular Boundary Method

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Abstract

The singular boundary method (SBM) is a relatively new strong-form collocation boundary discretization technology for the solution of boundary value problems. Unlike the method of fundamental solutions (MFS), the source and collocation points of the SBM coincide on the physical boundary without the requirement of fictitious boundary. The key idea in the SBM is to introduce the concept of origin intensity factor to isolate the singularity of fundamental solution. And if the problem of interest is mathematically well-posed, the fundamental assumption is the existence of the origin intensity factor upon the singularity of the coincident source-collocation nodes.

This method proposes an inverse interpolation technique to evaluate the singular diagonal elements of the interpolation matrix. This method is mathematically simple, accurate, easy-to-program, and truly meshless. However, in order to carry out the inverse interpolation technique, the SBM places a cluster of sample nodes inside the physical domain. Our numerical experiments indicate that the accuracy of SBM solution, in some cases, may be sensitive to the placement of such sample nodes.

This paper proposes an improved SBM formulation to avoid the above-mentioned sample nodes in the traditional formulation via the desingularization of subtracting and adding-back technique widely used in the boundary element method (BEM). The new formulation keeps all merits of the SBM and makes the method more stable, accurate, efficient, and extends its applicability. The present SBM formulation is tested to a few benchmark 2D potential problems in comparison with the other boundary discretization methods such as the MFS, the BEM and the boundary distributed source method.

Keywords: Singular boundary method, fundamental solution, singularity, desingularization technique, origin intensity factor.

Material Point Method with RBF Interpolation

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Abstract

This paper makes the first attempt to employ Radial Basis Functions (RBF) interpolation in the material point method (MPM), whose shape function is based on RBF and polynomial function and satisfies the partition of unity and possesses Delta-function property. It is worthy of stressing that RBF interpolation has merit of high smoothness and is very accurate and can easily be applied to MPM framework for mapping information between moving particles, known as material point in the MPM, and background grids. The RBF-based MPM is expected to overcome the unphysical results in the traditional MPM thanks to the application of linear shape function. This study chooses the Mutiquadric (MQ) function as the RBF. The present strategy is tested to the benchmark cantilever beam and the circular disk impact problems. Numerical results are observed in good agreement with the analytical ones and satisfy the conservation of energy.

Keywords: Material point method, Radial basis function, Multiquadric, interpolation, Meshless, Stress analysis

The connection between MFS and RBF interpolation

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Abstract

In this talk we present the connection between interpolation with the MFS method in dimension *D*+1 and the RBF interpolation in dimension *D*, connecting several examples of known RBF basis functions and their equivalents as fundamental solutions.

An implementation of the method of fundamental solutions for the dynamics of a plate large displacement

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Abstract

The nonlinear, dynamics of a plate large deflection is a subject of investigations across many disciplines. Various models have been proposed; among the most frequently encountered are those related to that of von Karman. In this case the initial-boundary value problem of the dynamics of a plate large deflection is described by two nonlinear, coupled partially differential equations of fourth order with two initial conditions and four boundary conditions at each boundary point.

There are various means of simulating the dynamic response of thin nonlinear plate, to various degrees of accuracy. The proposal of this paper is to implement one of the meshfree method i.e. the Method of Fundamental Solutions. The problem is solved in discretised time domain. This discretisation is done in a conception of the Finite Differences Method. The nonlinearity of the equations, obtained at each time step, is solved by application of Picard iterations. For each iterations step a boundary value problem is to solve. Moreover, the equations at each iteration step are inhomogeneous ones. So, the approximation by Radial Basis Function is applied and a particular solution of boundary value problems is obtained. The final solution is calculated by implementation of the Method of Fundamental Solution.

The numerical experiment has confirmed that the proposed numerical procedure gives the solutions with demanded accuracy and is good tool to solve the considered problem.

Keywords: the method of fundamental solutions, approximation by radial basis function, Picard iterations, dynamics of a plate large deflection, von Karman plate equations

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Method of fundamental solutions for the vibroacoustic analysis

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Abstract

The method of fundamental solutions, one of the promising boundary-type meshless methods, is proposed as a direct procedure to formulate and analyze the vibroacoustic problem. The coupled system discussed in this study is composed of an acoustic-cavity and excited by an external force or an internal sound source harmonically. The wall of cavity is composed of the beam or the plate components, respectively, in two- and three-dimensional problems. The two independent sub-systems interact at the interface simultaneously by satisfying the necessary equilibrium and compatibility conditions. The mathematical formulations described by the presented meshless method straightforwardly enable us to investigate the frequency responses of the vibroacoustic problem with no boundary integrals. General characteristics of the dynamic coupling effect are displayed, based on the systematic natural frequencies and mode shapes. Feasible results simulated by the presented numerical scheme are validated through numerical experiments including the acoustic-wave propagation problems and the vibroacoustic problems.

Keywords: method of fundamental solutions, vibroacoustic, natural frequency, mode shape, acoustic-wave propagation.

Parallel Section 12

On the exponential convergence of method of fundamental solutions

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Abstract

It is well known that the method of fundamental solutions (MFS) is an numerical method of exponential convergence. In other words, the logarithmic error is proportional to the node number of spatial discretization. In this study, the exponential convergence of MFS is demonstrated by solving Laplace equation in domains of rectangles, ellipses, amoeba-like shapes, and rectangular cuboids. In the solution procedure, the sources of the MFS are located as far as possible and the instability resulted from the ill-conditioning of system matrix is avoided by using the multiple precision floating-point reliable (MPFR) library. The results show that the convergence are faster for the cases of smoother boundary conditions and larger area/perimeter ratios. For problems with discontinuous boundary data, the exponential convergence is also accomplished using the enriched method of fundamental solutions (EMFS), which is constructed by the fundamental solutions and singular solutions.

Keywords: exponential convergence, method of fundamental solutions, corner singularity, multiple precision floating-point reliable library

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The Eulerian-Lagrangian method of fundamental solutions for the hyperbolic system problem

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Abstract

The objective of this paper is to develop a meshless numerical model for dealing with the system of hyperbolic equations. The proposed method based on the diffusion fundamental solution and the Eulerian-Lagrangian method is able to easily handle the hyperbolic equations. First we transfer the coupled hyperbolic system to the system of the decoupled pure-advection equations, and then approximate the solutions with the proposed meshless numerical scheme. The proposed numerical method is free from mesh, numerical quadrature and can easily transform the physical variables between the Eulerian and Lagrangian coordinates. There are some numerical tests for validating the proposed numerical scheme and the numerical results compared well with the analytical solutions. The problem of the linear shallow water equations is analyzed by the proposed meshless method and the results are compared well with the analytical solutions. Therefore the proposed model is a promising numerical solver for the hyperbolic system.

Keywords: Meshless numerical method, method of fundamental solutions, hyperbolic system, shallow water wave.

On the shape parameter of the MFS-MPS scheme

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Abstract

In this paper, we apply method of particular solution (MPS) and one-stage method of fundamental solution (MFS-MPS) for solving partial differential equation. In the 1-D Poisson equation, we prove that the solution of MFS-MPS converges to that of Spectral Collocation Method with Polynomial (SCMP), and show that the numerical solution using MFS-MPS is similar to the ones using MPS, Kansa's method, and SCMP.

Keywords: spectral collocation method, method of fundamental solution, particular solution.

Interaction of water waves with vertical cylinder using the method of fundamental solutions

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Abstract

This paper proposes applications of the method of fundamental solutions (MFS) to solve the scattering of water waves by bottom-mounted vertical circular cylinder. By using the two-point function of fundamental solutions, the coefficients of influence matrices are easily determined. It is found that this method also produces irregular frequencies as well as the boundary element method does. The position of the irregular frequency depends on the source point location. To avoid this numerical instability, the mixed-layer potential method is employed to deal with the problem. Based on the circulant properties and degenerate kernels, an analytical scheme in the discrete system of a cylinder is achieved to demonstrate the existence of irregular frequencies. One numerical example of scattering problem of a circular cylinder was examined and are compared with the results by using direct BEM.

Keywords: method of fundamental solutions, water wave, two-point function, irregular frequency, circulant, degenerate kernels.

Parallel Section 13

Regularization techniques for the method of fundamental solutions

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Abstract

The Method of Fundamental Solutions (MFS), which can be regarded a meshless version of the (indirect) Boundary Element Method, exhibits a lot of numerical advantages. First, it is a truly meshless technique, i.e. it requires neither domain nor boundary element structure; only a finite set of unstructured points is necessary. Next, it requires no sophisticated difference schemes to approximate the appearing derivatives (e.g. upwind discretization of the convective derivative in transport problems); also, there is no need to use carefully defined elements for discretizing the pressure and the velocities when solving the Stokes problem etc. However, a scattered data interpolation is often required. Using the popular globally supported radial basis functions (e.g. multiquadrics, inverse multiquadrics, thin plate or polyharmonic splines), this leads to a subproblem with a high computational cost, if the number of the base points of the interpolation is large. To reduce the computational cost, special techniques (e.g. domain decomposition) have to be applied. On the other hand, in the traditional version of the Method of Fundamental Solutions, some source points lying outside of the domain of the partial differential equation have to be introduced, due to the singularity of the applied fundamental solution at the origin. If these source points are located far from the boundary, the resulting system of algebraic equations becomes extremely ill-conditioned, which can destroy the numerical efficiency.

In this paper, a special kind of regularization techniques is applied. The partial differential operator appearing in the original problem is approximated by a higher order singularly perturbed operator with a carefully chosen scaling parameter. This new partial differential operator is (at least) of fourth order, and its fundamental solution is continuous at the origin. Using the fundamental solution of this new operator instead of the original one, the source and the collocation points are allowed to coincide. In this way, the use of extremely ill-conditioned systems can be avoided. Thus, a classical second order problem (e.g. the Poisson equation, the modified Helmholtz equation, as well as the convection-diffusion equation or also the classical Stokes equations) can be converted to another problem in which the operator is a fourth order multi-elliptic operator, but has a continuous fundamental solution. In some cases, the solution of the resulting linear equations can be completely avoided by using multi-elliptic interpolation and quadtree-based multigrid tools. These techniques are useful also in creating particular solutions. The applicability of the approach is demonstrated via several numerical examples.

Keywords: method of fundamental solutions, regularization, multi-elliptic operator, multi-elliptic interpolation, Stokes equations, quadtree, multigrid

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An average source meshless method for solving the potential problems

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Abstract

A new boundary-type meshless method, to be called the average source meshless method (ASMM), is presented in this paper that is easy to implement and has no requirement of the uniform distribution of boundary nodes. The method is based on an average source technique (AST) and the regularized boundary integral equations (RBIEs). By using the RBIEs to avoid the singularity of the kernel functions, the main difficulty of the coincidence of the source and collocation points disappears and therefore, unlike the traditional MFS, the source points can be located on the physical boundary. By using the AST the distributed source on a segment can be reduced to the concentrated point source and so the boundary integrals are generally not necessary. On the other hand, as we all known, the treatment of the boundary layer effect is a very difficult problem in any meshless method. Herein, we thoroughly solve the problem by AST. Furthermore, the proposed method can effectively compute any boundary fluxes $\partial u/\partial x_i$ $(i=1,2)$, but are only not limited to normal flux $\partial u/\partial n$. The accuracy, stability, efficiency and widely practical applicability are verified in numerical experiments of the Dirichlet, and mix-type boundary conditions of interior and exterior problem with simple and complicated boundaries. Good agreements with exact solutions are shown.

Investigation on Nearly-Boundary Solutions by Singular Boundary Method

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Abstract

The fundamental solution of governing equations in physics and mechanics encounters so-called singularity at origin. Not surprisingly, the traditional view is that the fundamental solution can not be used as the basis function in the numerical solution of a partial differential equation, except that the source nodes are placed on the fictitious boundary outside the physical domain and are separated from the collocation nodes on the physical boundary, which underlies the method of fundamental solution, a popular method in recent years. The singular boundary method (SBM) makes a breakthrough on this traditional view in that the method simply uses the fundamental solution as the interpolation basis function while keeping the same source and collocation nodes on the physical boundary. The fundamental assumption is the existence of the origin intensity factor upon the singularity of the coincident source-collocation nodes.

The accurate computation of the kernel functions plays an important role in the implementation of the SBM. When the collocation point is far from the boundary under consideration, a straightforward application of the SBM expansion is sufficient to obtain accurate numerical results. However, if the collocation point is close to, but not on, the boundary, the 'nearly' singular problem will arise in the conventional SBM formulation. Similar to the singular problem, the near singularity comes from the properties of the fundamental solutions and the respective derivatives.

Nearly singular kernels are not singular in the sense of mathematics. However, from the point of view of numerical calculation, these kernels can not be accurately calculated in the conventional SBM approach since the kernel oscillates seriously as the collocation point moves closer to the boundary, similar to the so-called boundary layer effect in the boundary element method.

The purpose of this paper is to develop a general strategy suitable for a wide range of nearly singular kernels. An inverse interpolation technique is employed to remove or damp out the near singularity when the collocation point gets closer to the boundary. By using the proposed strategy, we can effectively handle the seemingly difficult task of evaluating the nearly singular kernels in the SBM formulation.

Keywords: Singular boundary method, fundamental solution, origin intensity factor, near singularity, inverse interpolation technique.

Solving the direct and inverse Stokes problems by the boundary knot method and Laplacian decomposition

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Abstract

A meshfree numerical scheme based on the boundary knot method (BKM) is proposed for the two-dimensional direct and inverse Stokes problems, which is highly ill-conditioned. Due to the Laplacian decomposition, the coupled Stokes equations are converted to three Laplace equations. Then the BKM, which is a boundary-type meshless collocation scheme, is used to solve these three Laplace equations. In comparing with the method of fundamental solutions, the sources in BKM are located along the physical boundary to avoid the choice of the source positions. In BKM, the numerical solutions of these three Laplace equations are expressed by nonsingular general solution of two-dimensional Laplace equations. The unknown coefficients in the solution expressions are then found by satisfying the boundary conditions at the boundary collocation points. Several numerical examples will be provided to demonstrate the efficacy and stability of the proposed scheme for solving the direct and inverse Stokes problems.

Keywords: boundary knot method, Stokes problem, Laplacian decomposition, boundary-type meshless collocation method

The boundary knot method for Poisson and inhomogeneous biharmonic problems

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[2]State Key Laboratory of Structural Analysis for Industrial Equipment
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Abstract

The boundary knot method (BKM) is a recent meshfree boundary collocation technology. The key idea in the BKM is to introduce the nonsingular general solutions instead of the singular fundamental solutions to avoid the singularity at the origin. However, the BKM encounters troubles that the nonsingular general solution is not available for problems with harmonic operators. To overcome the above-mentioned barrier in the BKM, we proposed a nonsingular harmonic solution for approximation of Poisson and inhomogeneous biharmonic problems. In the proposed strategy, the dual reciprocity method (DRM) is employed to evaluate the particular solution in inhomogeneous problems, and we combine the original two-step BKM-DRM into one-step algorithm， which avoid the round-off errors between the two steps. The regularization technique is utilized to remedy detrimental effects from ill-conditioned interpolation matrices accompany with large number of boundary nodes. The feasibility and efficiency of the proposed BKM formulation with the nonsingular harmonic solution are verified through several benchmark numerical examples, including isotropic problems and anisotropic problems in both two dimensions and three dimensions.

Keywords: boundary knot method, Poisson equation, inhomogeneous biharmonic equation, nonsingular harmonic solution, dual reciprocity method, anisotropic problems

Heat conduction analysis in functionally graded materials by two boundary collocation methods

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Abstract

This paper presents a nonsingular general solution and T-complete functions of heat conduction in nonlinear functionally graded materials (FGMs). The solutions is then used to derive the boundary knot method (BKM) and collocation Trefftz method (CTM) in conjunction with Kirchhoff transformation and various variable transformations in the solution of nonlinear FGM problems. Further, Laplace transform technique is used to handle the time variable and the Stehfest numerical Laplace inversion is applied to retrieve time-dependent solutions. The proposed BKM and CTM are mathematically simple, easy-to-program, meshless, high accurate and integration-free, and avoid the controversial fictitious boundary in the method of fundamental solution (MFS). Three numerical examples are considered and the results are compared with those from MFS, LBIEM and analytical solutions for demonstrating the efficiency and accuracy of the present schemes in the solution of steady state and transient heat conduction in nonlinear FGMs.

Keywords: Boundary knot method, collocation Trefftz method, Kirchhoff transformation, Laplace transformation, heat conduction, meshless

Parallel Section 14

2D shallow water equations by localized meshless methods

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Abstract

In this paper, localized meshless numerical methods using radial basis function differential quadric (LRBFDQ) method is applied to solve two dimensional shallow water equations. Generally speaking, meshless (meshfree) methods can be easily applied to problems with complex boundary shapes. The LRBFDQ method, a newly developed meshless method, has been found to have high engineering applicability, because its weighting coefficients can be determined by different radial basis functions (RBFs) depending on the interested problems. With different characteristics, the adopted RBFs can be chosen from several types such as multi-quadric (MQ), integral MQ, Gaussian, polyharmonic spline, or integral polyharmonic spline. For the numerical experiments, the 2D dam break problem has been solved first, then some applications in real project such as the Taiwan river system will be shown in the text. The analyses of the numerical results by different RBFs will be discussed in details and then compared with analytical and other numerical solutions in the literature. By this study, the performance of this meshless numerical method is demonstrated, and the most appropriate RBFs for 2D shallow water equations is obtained.

Keywords: RBF, LRBFDQ, shallow water equations, meshless, 2D, MQ, Gaussian, polyharmonic spline

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The local radial basis function differential quadrature method for 1D shallow water equations

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National Taiwan University, Taipei 10617, Taiwan
E-mail: dlyoung@ntu.edu.tw

Abstract

In this paper, the local radial basis function differential quadrature (LRBFDQ)

method is applied to solve the shallow water equations. Meshless methods have been suggested to solve hydraulic problems. The LRBFDQ method is one of the new developed meshless methods. This localized approach is developed from the differential quadrature (DQ) method by employing the radial basis function (RBF) for the test function. Two hydraulic engineering cases are demonstrated for numerical analyses. First, a dam break problem is adopted to verify the accuracy of this procedure. There is consistency between the numerical results and the analytical solutions. And the other case is the simulation of the inflow of the Yuanshantze Flood Diversion. The solutions by LRBFDQ are well corresponded with the HEC-RAS model. According to the numerical results it indicates that the LRBFDQ method is stable, accurate and robust for solving the shallow water equation problems.
Keywords: shallow water equations, LRBFDQ, DQ, RBF, meshless.

Local radial basis function-based differential quadrature method for 2-D free surface problem

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Abstract

The approach of differential quadrature combined with the radial basis function by using local collocation to simulate the 2D free surface investigations have been done in present study. Local radial basis function-based differential quadrature (RBF-DQ) method can be considering a mesh-free numerical scheme because the trial function of the local RBF-DQ is replaced with a RBF. Present method not only retains the high accuracy also has the capability of applying to the moving boundary problem. These advantages can directly improve the computational efficiency and save further the memory of mechanic when dealing with the free surface problems. Furthermore, the numerical results have satisfied the good agreement with the existing solution for 1D demonstration and traditional numerical scheme such as finite element method (FEM), experimental measurements and other numerical results. The validation of current numerical simulation was shown the expected results that overcame the issue of mesh distortion for caused by the traditional mesh type numerical means involving the structured and un-structured mesh grid. Therefore, the proposed numerical model provides the promising means to solve the troublesome problem of free surface.

Keywords: approach of differential quadrature, radial basis function, local collocation, 2D free surface

Pricing options for the jump-diffusion models by the local differential quadrature method

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Abstract

In this paper the local differential quadrature (LDQ) method is provided to solve the option-pricing problems based on jump-diffusion models. The main advantages of the LDQ method include the arbitrary order for the numerical approach and the convenience to deal with non-uniform grids. For option-pricing works, jump-diffusion processes are widely used to simulate the variation of asset prices. Generally speaking, the pricing equation is a partial integro-differential equation (PIDE) in jump-diffusion models. Thus, how to solve the PIDE efficiently is an essential topic for financial engineers. For this purpose, the LDQ method dealing with non-uniform grids is adopted. Moreover, an alternative integral method appropriate for non-uniform grids is also suggested. Numerical analyses for European, American and barrier options are demonstrated in this work. As a result, the LDQ method combined with the alternative integral method provides good performance for solving the jump-diffusion models. It also indicates that the option-pricing works can be progressed more efficiently by the present method.

Keywords: option-pricing, jump-diffusion models, LDQ, PIDE, European option, American option, barrier option

The interpolation techniques based on the local radial basis function differential quadrature method

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Abstract

An interpolation technique based on the local differential quadrature (LDQ) method is proposed to interpolate the unknown data value with arbitrarily scattered given data. By employing the multiquadric radial basis functions (MQ) as test functions, the LDQ method is a mesh-free numerical scheme with high accuracy. In this study, the unknown data are interpolated with the help of the field gradients, which take care of the geometric properties, and the governing equations, which forces the interpolated data to satisfy the physical principles. Further, an optimum scheme that combines the merits of both is also present. Three-dimensional numerical examples governed respectively by the Poisson and theadvection-diffusion equations are performed to validate the accuracy and the stability of the interpolation technique. The present results, comparing with the results by the linear polynomial fitting (LPF) method and the quadratic polynomialfitting (QPF) method, shows that the present interpolation technique is more accurate and robust than conventional ones.

Keywords: interpolation, meshless numerical method, local differential quadrature, radial basis functions (RBFs), advection-diffusion equations

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A linear iterative investigation for MFS with EEM to solve the 3D nonhomogeneous diffusion equation

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Abstract

The eigenfunctions expansion method (EEM) will first introduce in the present study to combine the method of fundamental solutions (MFS) and the method of particular solutions (MPS) to get the numerical solutions of 3-D diffusion equation with nonhomogeneous type. Proposed meshless numerical model named the model of

MFS-MPS-EEM are adopted further to analyze the nonhomogeneous diffusion equation in complex domain corresponding the boundary conditions. For complex domain, the nonhomogeneous diffusion equation can divide individually into a Poisson equation and a homogeneous diffusion equation through the presented numerical model. In which the Poisson equation was calculated by the MFS-MPS model where the compactly-supported radial basis functions has direct adopted for the MPS and the fundamental solution of Laplace operator has used for the MFS. Furthermore, in terms of homogeneous diffusion equation was translated into a Helmholtz equation by EEM, which can be solved using the MFS combined with the singular value decomposition (SVD). Good agreement with the analytical and finite element solutions was obtained; hence, the present numerical scheme has shown a promising mesh-free numerical tool to solve the 3-D nonhomogeneous diffusion equations with time-independent source terms and boundary conditions.

Keywords: method of fundamental solutions, method of particular solutions, eigenfunctions expansion method, 3-D nonhomogeneous diffusion equation, singular value decomposition

Accommodation

Exchange Rate on July, 2010 is around USD:NTD=1:32, please note that the room rates to USD conversion below is only for your reference, please confirm with the hotel of your choice when you make reservation.

Hotel	Room Type	Room Rates			
		Single	Twin	Triple	Family
Kingship Hotel	Standard Room	NT$ 1,600	NT$ 1,800	NT$ 2,400	NT$ 3,000
		N/A	N/A	N/A	N/A
	**Kingship Hotel** is a 4-star business hotel in the elegant European style building, only 3 minutes walking distance away from O2 stop of MRT (Orange Line of Kaohsiung Mass Rapid Transit). Kingship has 228 rooms, 4 restaurants and a fine collection of 250 oil paintings mostly by Taiwanese painters. It is easy to reach variety of fine shops, shopping centers, and department stores, and enjoy local or traditional food near Kingship.				
No.98, Cisian 3rd Rd., Kaohsiung city, Taiwan. http://en.kingship.com.tw/					

Hotel	Room Type	Room Rates			
		Single	Twin	Triple	Family
Zhong-Zheng Hotel	Superior Room	NT$ 580	NT$680	N/A	NT$ 880
	Zhong-Zheng Hotel				
No.7, Daren Rd., Kaohsiung City, Taiwan http://www.kha.org.tw/webs/hotel7/post.aspx?menuID=465					

Remarks

1. 10% service charge and 5% tax are included in the above rates.
2. Airport transport to hotel could be extra ordered. (Reservation must be made 7 days prior to your arrival date).

Sponsors

1. National Science Council (http://web1.nsc.gov.tw/mp.aspx)

2. National Science Council - Mathematics Research Promotion Center (http://proj1.sinica.edu.tw/~mrpcwww/)

3. National Sun Yat-sen University (http://www.nsysu.edu.tw/bin/home.php)

4. NTOU/MSV (http://msvlab.hre.ntou.edu.tw/index1.htm)

5. Scientific Formosa, INC. (http://www.sciformosa.com.tw/)

6. The Numerical Algorithms Group — NAG (http://tw.nag-gc.com/)

7. Intelligent Data Miner Inc. (http://www.idminer.com.tw/)

8. TeraSoft Inc. (http://www.terasoft.com.tw/)

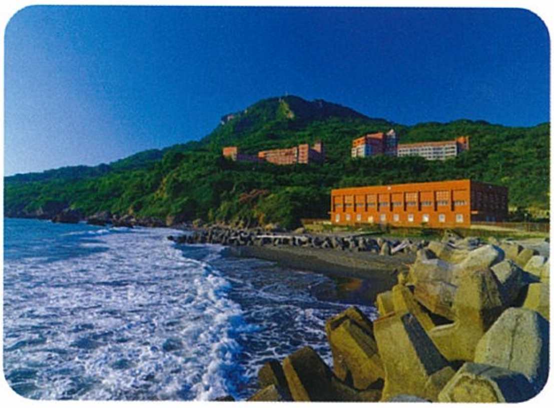
Author index

National Sun Yat-sen University
Kaohsiung, Taiwan

University Profile

National Sun Yat-sen University was established in Kaohsiung city in 1980, and was named in honor of Dr. Sun Yat-sen, the founding father of the Republic of China. With the dedicated efforts of faculty, staff, and students, NSYSU has quickly developed into one of the top universities in Taiwan. Presently, the University consists of six colleges, including Liberal Arts, Science, Engineering, Management, Marine Sciences, and Social Sciences, and offers 20 Bachelor, 37 Master and 27 Doctoral programs. There are currently 466 full-time faculty members and 9,675 students.

Rankings and Accreditation



International Rankings

- 2010 QS Asian University Ranking
  7th in Taiwan, 62nd in Asia
- 2009 QS World University Ranking
  401-500 category
- 2009 Webometrics Ranking of World Universities
  3rd in Taiwan, 124th in the world
- 2009 Ranked 48th in the world by Financial Times in Global EMBA and 2nd in Taiwan
- 2009 Ranked 48th in the world by Financial Times in "Global Masters in Management" and 2nd in Taiwan.

International Accreditation

- 2005 College of Management, the first in Taiwan to be accredited by AACSB
- 2010 College of Management, the first in Taiwan to be reaccredited by AACSB
- 2005 College of Engineering received accreditation from IEET (Signatory of Washington Accord)
- 2010 College of Engineering reaccredited by IEET
- 2010 Dept. of Marine Environment and Engineering received IEET Accreditation

Key Research Areas

As a medium-sized research university, it is essential that NSYSU promotes the research capacity of our faculty and exercises the function of interdisciplinary research centers. With the special funding of "Aim for the Top University Plan" from the Ministry of Education, and the privileged location situated by the ocean, NSYSU has achieved great success in research in the fields of marine sciences, management, economic, opto-electronics, micro/nano technology, and wireless communication, which is reflected on the high number of publications in international academic journals that our faculty contributed in the recent years.

Balancing between Humanities and Sciences: 3 major research groups

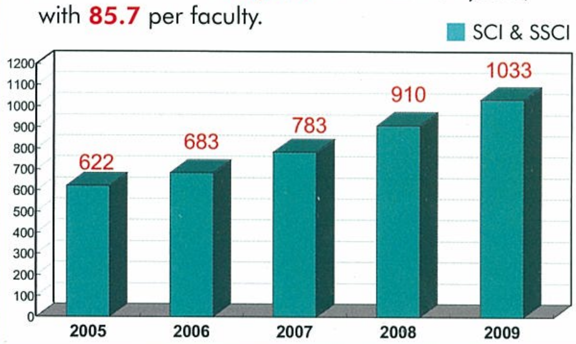
In considering the overall development and the areas of humanities and sciences, the research centers are classified into three major groups:

Elite Research Group	Key Research Group	Foundational Research Group
Asia-Pacific Ocean Research Center	Opto-electronics Center of Excellence	Center for the Humanities and Social Sciences
Electronic Commerce and Technology Innovation Research Center	Center for Nano Science & Nano Technology	National Policy Research Center
Wireless Communication Antenna Research Center	NSYSU-KMU Joint Research Center	Research Center for Nonlinear Analysis and Discrete Mathematics

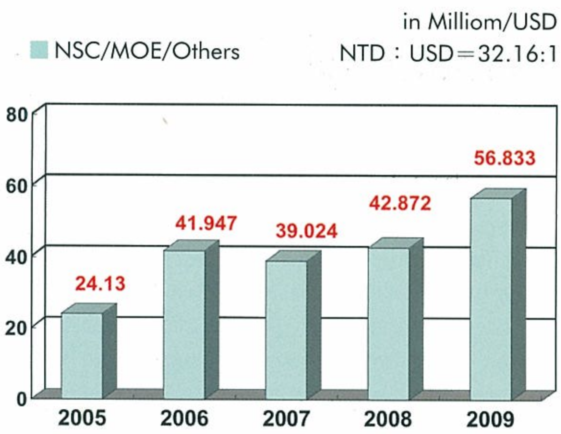
Academic Accomplishments

SCI / SSCI Papers

- Number of Papers published per faculty: **2.2**
- Number of SCI papers published for science and engineering faculty: **4.0**
- Citations reached **40,714** in the last ten years, with **85.7** per faculty.



Total Funding for Research



Nurture Future Leaders

- Diverse programs including "hands-on experience" and "voluntary leadership" to equip students to be leaders of the future.
- Funding for student participating in international competitions:

2007 Gold medal at the "2007 World Roller Speed Skating Championships" Men's 3,000 -meter Relay Final

2007 Second Place in the 2007 L'Oreal E-strat Challenge

2009 4 Gold medals and 3 Silver medals in Speed Skating at the 2009 World Games

- Talented human resources are the foundation of excellence in academic research. In addition to the grants from National Science Council for research projects, the faculty also benefit from the flexible salary package, sufficient research funding, and quality research environment. NSYSU formulates substantial objectives to recruit outstanding young professors, contract professors, chair professors, visiting professors, postdoctoral researchers, and excellent graduate students to form a strong and highly motivated research team.

Create a First-class Learning Environment

- **Ecological Campus**

  Situated by the beautiful beach and mountains, NSYSU provides a great environment for not only the students and faculties, non-dangerous wildlife can be seen on campus, such as monkeys, cockatoos, and squirrels.

- **Digital Society**

  The "Learning Plaza" in the library provides diverse learning facilities including Discussion & Presentation rooms, Digital information resource section, and seminar rooms and reading rooms. The Library also expanded its collection to over 700,000 Chinese and English books and periodicals, and more than 1,700,000 electronic books, and established an "e-book collection" to provide a user-friendly digital learning environment.

- **Culture Space**

  NSYSU holds Sunshine Art Festival and Hsi-wan Performing Arts Festival every year. Talented overseas and domestic professional individuals and groups are invited to perform at the art festivals. NSYSU is also the first university in Taiwan that establishes a resident orchestra on campus. The Baroque Camerata is founded in 2004 by 18 talented faculty members and students from the Department of Music of NSYSU.

International Collaborations

Measurements in creating an international campus

NSYSU is currently emphasizing on the recruitment of Talented international research and teaching professionals, Joint-supervised Ph.D. Students, Post-doctoral research Fellow, and Short-term teaching staff. Thus, the university hopes to increase the quality of research and teaching, the number of international collaboration programs, the Dual-degree program, Visiting Student Program, and to further enhance the global vision and competitiveness of students at NSYSU.

International Exchange and Degree student Program

An important sign for internationalization of a campus is the number of international students. Every year we receive more than 150 exchange students from our sister universities. And in order to attract outstanding international students to come to study at NSYSU, various types of scholarships are available for degree-seeking students.

2010 International Degree Students: **298**

Inbound Exchange Students: **197**

Outbound Exchange Students: **153**

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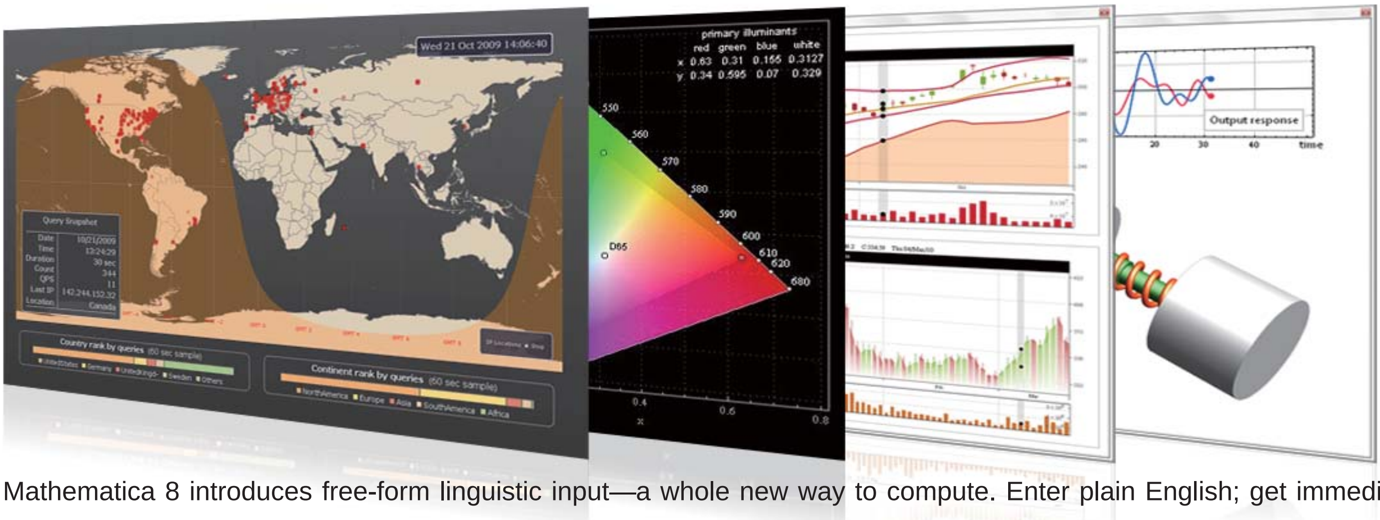

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Top 8 reasons to upgrade to *Mathematica*® 8

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6. Use more powerful image processing and analysis capabilities
7. Create interactive tools for rapid exploration of your ideas
8. Develop faster and more powerful applications

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By using NAG Library algorithms you cut key person dependency inherent if you choose to write your own code. The NAG Library is continually being updated and improved.

Detailed documentation

NAG's documentation is renowned for its detail. Included in each NAG Library function document is an example program giving users a template for adaptation to their own problems.

NAG's expert support service

By subscribing to NAG's dedicated in-house Customer Support Service not only will you receive product updates which includes new and improved algorithmic functionality, but you can contact NAG experts who will assist with your technical queries or difficulties.

Features		Benefits
Unrivalled numerical functionality – over 1,600 routines	→	Save time by not writing your own algorithmic code
Proven correctness and robustness	→	Future proof accuracy of application and reduce maintenance time
Highly flexible and easy to use from multiple languages and environments	→	NAG software evolves to reflect your programming language and environment advances
Thread safe routines	→	Safely develop multi threaded applications

Product availability

The NAG Library is available for: Linux, Microsoft Windows, Mac OS and many more, and is callable from multiple software packages, programming languages and development environments.

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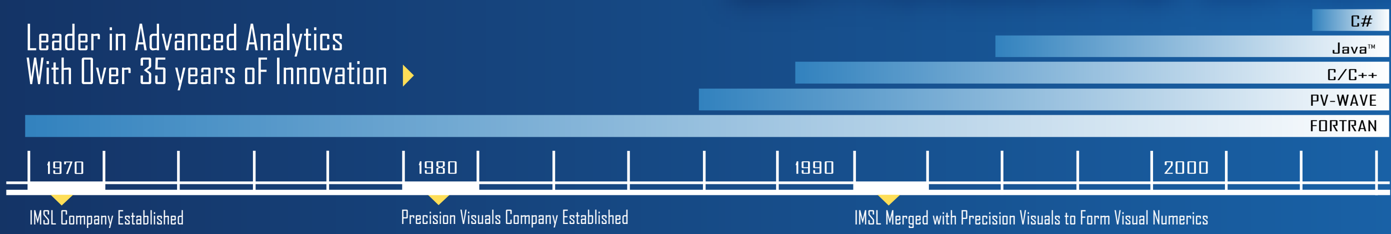


Complementary Roles Hardware libraries strong in utilities and foundation algorithms
IMSL™ Library strong in extended mathematics and statistics algorithms

	Algorithm Area	IMSL Fortran Library	Hardware Library
Extended Math and Statistics	Optimization	✓	Limited, if any
	Probability Distribution Functions, Inverses	✓	Limited, if any
	Special Functions	✓	Limited, if any
	Time Series / Forecasting	✓	
	Correlation	✓	
	Regression	✓	
	Foundation Statistics	✓	
	Cluster Analysis	✓	
	Factor and Discriminant Analysis	✓	
	Multidimentional Scaling	✓	
	Discriminant Analysis	✓	
	Categorical / Discrete Analysis	✓	
	Survival Analysis, Life Testing, Reliability	✓	
	Density and Hazard Estimation	✓	
	Analysis of Variance	✓	
	Sampling	✓	
	Tests of Goodness-of-Fit and Randomness	✓	
	Nonparametric Statistics	✓	
	Sorting and searching	✓	Limited, if any
	Nonlinear Equations	✓	Limited, if any
	Differential Equations	✓	Sometimes
	Quadrature	✓	Sometimes
	Interpolation	✓	Sometimes
	Random Number	✓	Limited, if any
	Vector / Matrix Processing	✓	Sometimes
	Eigensystem	✓	Sometimes

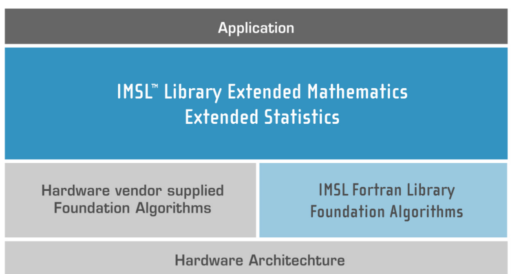
	Algorithm Area	IMSL Fortran Library	Hardware Library
Utilities	Matrix Operators	✓	Sometimes
	Box DAta Type Parallelization	✓	
	ScaLAPAC Error Checking	✓	Sometimes
	Error Handling	✓	Sometimes
Performance Foundation	FFT	✓	✓
	BLAS	✓	✓

IMSL plays a complementary role with the hardware supplied libraries.

The hardware libraries generally provide basic foundation library coverage.

IMSL leverages the hardware libraries and enables users to extend the benefits of their hardware investment by providing a full package of math and statistics functions with a focus around high performance and accuracy.

Even where IMSL and the vendor libraries share the same category, IMSL usually provides many more algorithms for the user, with over 1,000 FORTRAN functions available.



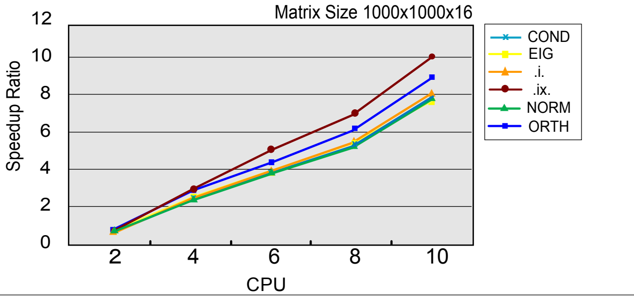
IMSL Parallel Computing Support

MPI Support

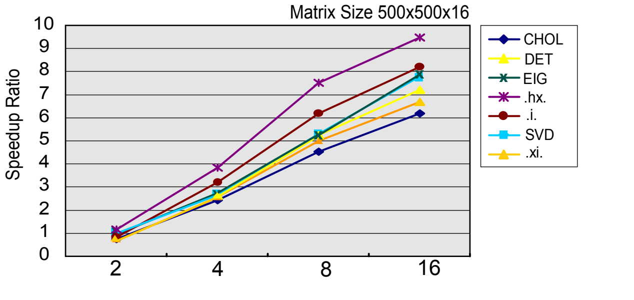
IMSL Fortran Numerical Library provides a dynamic interface for computing mathematical solutions over a distributed system via Message Passing Interface (MPI). MPI enabled routines offer a simple, reliable user interface for box data such as in linear systems, eigensystem analysis, transforms, and basic matrix and vector operations. IMSL also provides simple but valuable MPI utilities. Additionally, IMSL's ScaLAPACK utilities provide a convenient tool for MPI programming for ScaLAPACK use.

MPI Parallel Performance Examples

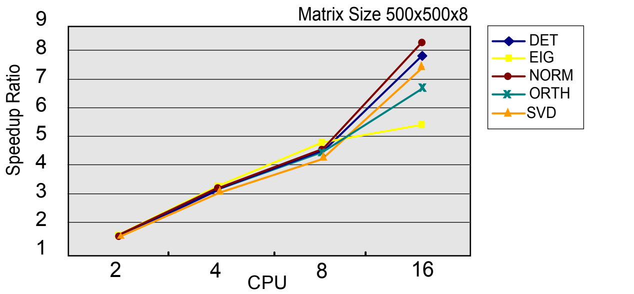
AMD Opteron SuSE Linux



IBM POWER5 AIX



Intel Itanium2 Red Hat Enterprise



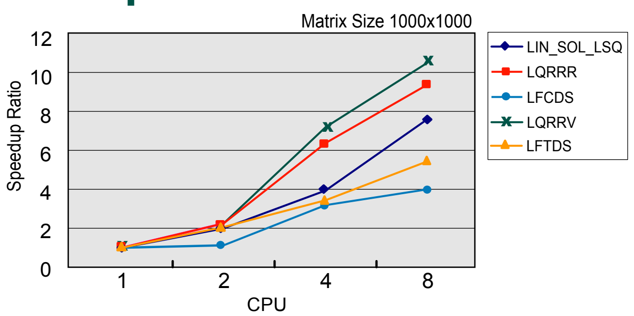
.hx. : Transpose of a matrix times a vector or a matrix
.i. : Inverse matrix or Moore-Penrose generalized inverse matrix
.ix. : Inverse matrix times a vector or a matrix
.xi. : A vector or a matrix times inverse matrix
CHOL : Cholesky factorization
COND : Condition number of a rectangular matrix
DET : Determinant of a rectangular matrix
EIG : Eigenvalue and eigenvector of ordinary or generalized eigenvalue problem
NORM : Norm of rank-1 or rank-2 array
ORTH : Orthogonalizes the columns of rank-2 or rank-3 array
SVD : Singular value decomposition of rank-2 or rank-3 array

OpenMP Support

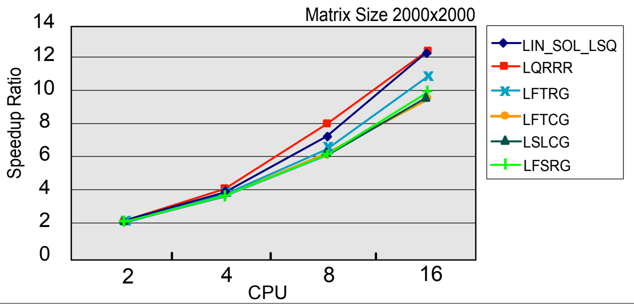
The IMSL Fortran Numerical Library offers expanded SMP support for a number of parallel processing environments. Computationally intensive algorithms in the area of linear systems, eigensystem analysis, transforms, random number generation and many others leverage SMP capabilities on a variety of systems.

OpenMP Parallel Performance Examples

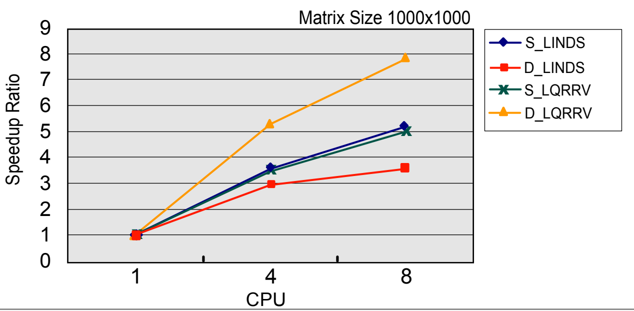
AMD Opteron SuSE Linux



IBM POWER5 AIX



Sun UltraSPARC Solaris



LFCDS : Cholesky factorization of a real symmetric matrix and condition number estimation
LFSRG : Solve a real general linear equations using LU factorization
LFTCG : LU factorization of a complex general matrix
LFTDS : Cholesky factorization of a real symmetric positive definite matrix
LFTRG : LU factorization of a real general matrix
LIN_SOL_LSQ : Solve linear equations by least square method
LINDS : Inverse of a real symmetric positive definite matrix
LQRRR : QR decomposition using Householder transformation
LQRRV : Least square solution in blocked form
LSLRG : Solve a real general linear equations

TIBCO Spotfire S+® Product Overview

Originally developed by Bell Laboratories, the S® programming language was the first language specifically developed for the creation of analytic prototypes. Analytic applications can be developed as much as five times faster than with other commercial languages or systems.

The Challenge
Despite widespread recognition of the importance of using predictive analytics and advanced statistics to analyze key business data and optimize business processes, many businesses make critical decisions based on analyses done with ad hoc and difficult to maintain spreadsheets. Others make decisions with out-of-date or irrelevant analytics, because it takes too long to create a useful predictive model and then deploy it into reports for decision makers, or to integrate it into key automated business processes.

The Solution
The Spotfire S+® product family provides statisticians and quantitative analysts with a broad array of predictive analytic and cutting-edge statistical analysis tools for powerful and effective data analysis, including statistical graphics that can rapidly convey unique insights into your data. These tools can be rapidly customized and extended to match the specific considerations and analytic requirements of your business. These analytic applications can then be rapidly deployed to provide insight to your business decision makers, and integrated into repeatable, validated processes within your organization.

Build Your Success on a Solid Foundation
Spotfire S+ software provides your organization with a comprehensive platform for creating and deploying the predictive analytics and advanced statistics solutions to build and grow your competitive edge. Spotfire S+ software contains the most comprehensive library of validated statistical algorithms available, and is readily supplemented with the Spotfire S+ package system (http://csan.insightful.com), which makes it easy to convert and use many packages from the open-source R system. Combined with an integrated development environment and an open architecture, Spotfire S+ software provides seamless and robust development, deployment and integration with your existing enterprise applications

Measure, Refine and Redeploy Quickly and Efficiently
Predict, execute and learn is the mantra of the high performance company. You are constantly measuring, evaluating and improving your performance. With the Spotfire S+ product family the lessons you learn are easily integrated to immediately improve your results. Where some commercial analytics products require weeks and months of laborious programming, Spotfire S+ software can prototype and deploy in hours or days.

From Prototype to Production Strength
The Spotfire S+ desktop products, Spotfire S+ and TIBCO Spotfire Miner™, enable your analysts and developers to quickly model and test analytic applications. Pull data from any networked resource and seamlessly deploy to the TIBCO Spotfire S+® Server for everyday production of regularly scheduled reports or produce answers on demand.

TIBCO® The Power of Now®

Spotfire®

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TIBCO Software Inc. (NASDAQ: TIBX) is the leading independent business integration software company in the world and a leading enabler of real-time business, helping companies become more cost-effective, more agile and more efficient. TIBCO has delivered the value of real-time business, what TIBCO calls The Power of Now®, to thousands of customers around the world and in a wide variety of industries.

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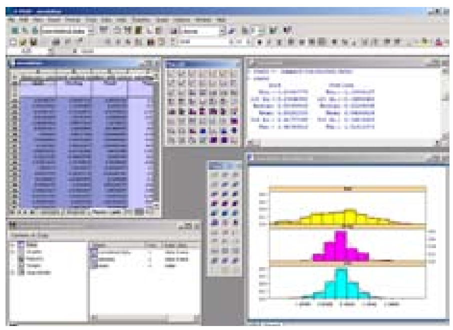
TIBCO®
The Power of Now®

Communicate Results Immediately and Effectively
Analysts can explore and communicate their findings with a broader base of business users by combining the power of predictive analytics and statistical modeling with interactive visual analysis capabilities of other TIBCO Spotfire products. Spotfire S+ and Spotfire Miner provide deep analytic insight into data through statistics and predictive modeling, which guides and focuses further analysis in products like TIBCO Spotfire Professional and TIBCO Spotfire Web Player.



Spotfire S+
Statistical analysis and programming environment for the desktop

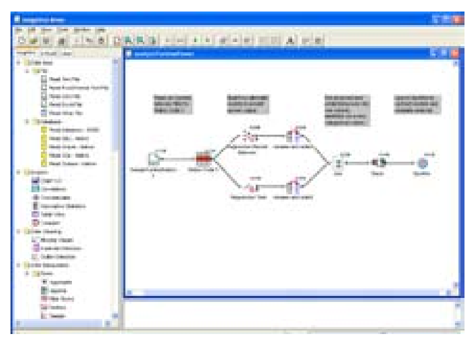
- Import and transform data from multiple sources including Excel, SAS and databases using powerful and flexible data preparation tools
- Rapidly prototype, develop and deploy new analytic applications using a broad array of data analysis and data mining functions
- Scale analysis on extremely large data sets on the desktop using Spotfire S+ software's big data library
- Share work and extend the statistical analysis palette by converting R packages using the Spotfire S+ package system
- Generate and automate custom reports using powerful statistical graphics and RTF or XML tools
- Deploy applications to the enterprise via TIBCO Spotfire S+ Server

Spotfire Miner™
Powerful and flexible data mining using intuitive graphical workflows

- Create documented, reusable analytic processes through visual workflow development for accountability, auditability and lower maintenance
- Extend and customize with new analytic methods developed with Spotfire S+
- Deploy analysis workflows to the enterprise using Spotfire S+ Server
- Scale analysis to very large data sets using unique pipeline processing

Spotfire S+ Server
Enterprise deployment and integration of advanced analytics

- Integrate analytics into client and web applications using C#/.NET or Java APIs
- Scale analysis to the largest data sets using unique pipeline processing
- Schedule and automate analysis with batch-processing or SOA integration into production environments
- Maintain security with single sign on authentication with LDAP or Microsoft Active Directory and encryption with SSL and HTTPS protocols

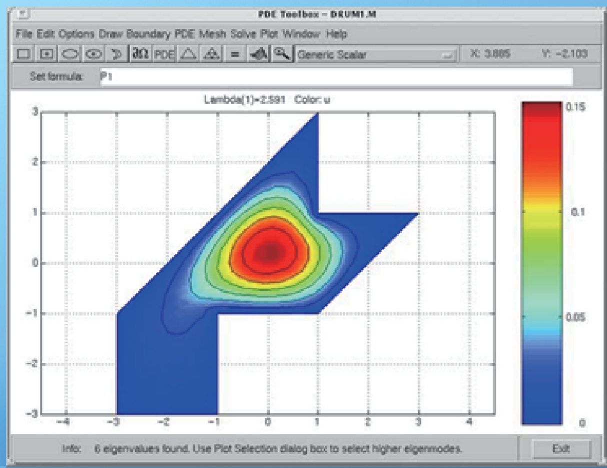


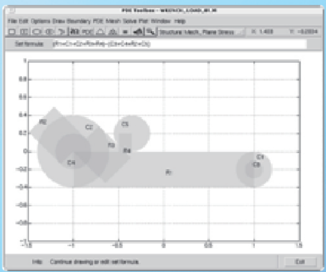
MATLAB®

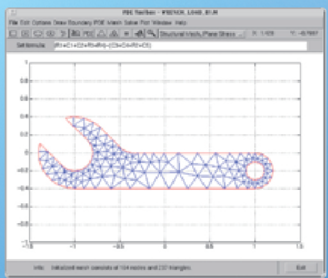
for Mathematical Modeling

Develop and optimize mathematical models of complex systems

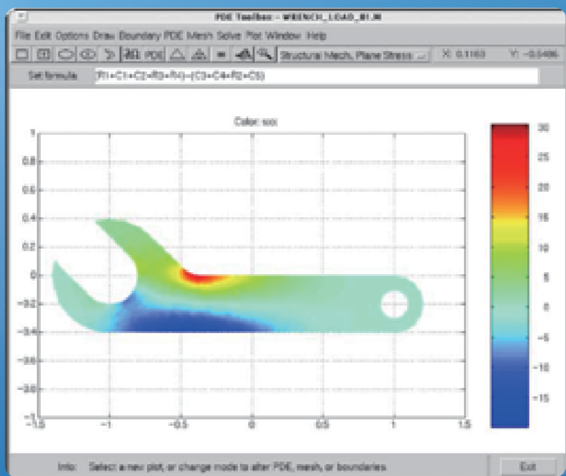
MathWorks products provide all the tools you need to develop mathematical models. MATLAB® supports both numeric and symbolic modeling approaches and provides curve fitting, statistics, optimization, ODE and PDE solving, calculus, and other core mathematical tools.

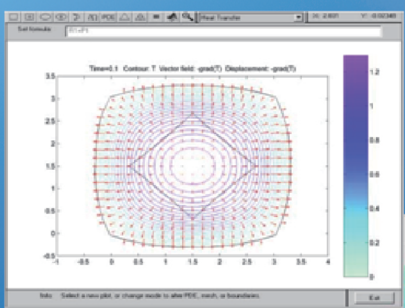


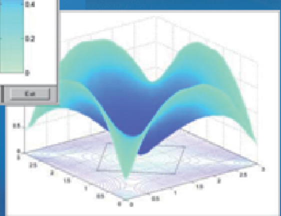




Using the graphical user interface to define the complex geometry of a wrench, generate a mesh, and analyze it for a given load configuration.







Visualization tools provide multiple ways to plot results. A contour plot with gradient arrows shows the temperature and heat flux. The temperature gradient is displayed using 3-D plotting tools.

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