Math 165: 1110 Exam 3

Name:_____

Section:_____

1. (a) Find the general antiderivative of the function: (8pt)

$$f(x) = 2x + \frac{3}{2}\sqrt{x} + 6e^{3x} - \pi \sin(\pi x)$$

[You don't need to simplify your answer, but do remember to add "+C".]

(b) Suppose $F'(x) = f(x)$ and $F(0) = 5$. Find $F(x)$. (2pt)
[Hint: Use $F(0) = 5$ to solve your C. Notice that e^0 is 1 but not 0.]

2. Use the L'Hôpital's rule or any method to find the following limits. (10pt)

(a) $\lim\limits_{x \to \infty} \dfrac{\ln x}{x} =$

(b) $\lim\limits_{x \to 3} \dfrac{x^2 - 4x + 3}{x^2 - x - 6} =$

3. (a) Find the derivative of 3^x.

[Use the fact $3^x = e^{\ln(3^x)} = e^{(\ln 3)x}$ and the fact $(e^u)' = e^u \cdot u'$. When you see $e^{(\ln 3)x}$ in your answer, replace it with 3^x, since it looks nicer.]

(b) Find the derivative of x^x.

[Use the fact $x^x = e^{\ln(x^x)} = e^{(\ln x)x}$ and the fact $(e^u)' = e^u \cdot u'$. When you see $e^{(\ln x)x}$ in your answer, replace it with x^x, since it looks nicer.]

4. Following the steps below to determine the absolute maximum value of $f(x) = x^3 - 3x^2 + 2$ on $[1, 3]$.

(a) Find $f'(x)$. (2pt)

[Double check your answer, since the whole problem relies on it.]

(b) Find the critical point(s) inside $[1, 3]$. (3pt)

(c) Find the boundary point(s). (2pt)

(d) Compare the values of $f(x)$ on these points to find the absolute maximum point. (3pt)

[Your answer should be of the form of $(x, f(x))$.]

5. Three points A, B, O are on a plane with $\overline{AO} = 3$, $\overline{BO} = 4$ and $OA \perp OB$. Car 1 starts at A driving toward O; Car 2 starts at B driving away from O. If at the beginning the speed of Car 1 is 1 and the rate of change of the distance between the two cars is 5, what is the speed of Car 2?

[You can use your own way to find the answer, below is one possible way to solve the question.]

(a) Let t be the time, starting at $t = 0$. Let $x(t)$ be the distance between Car 1 and O at time t, $y(t)$ the distance between Car 2 and O at time t, and $w(t)$ the distance between Car 1 and Car 2 at time t. What is the relation between $x(t)$, $y(t)$ and $w(t)$. (2pt)

(b) Base on what are given, find out $x(0)$, $y(0)$, $w(0)$, $x'(0)$, and $w'(0)$. (5pt) [Be aware of the direction for $x'(0)$.]

(c) The question is asking for $y'(0)$. Take the derivative (with respect to t) on the both sides of the relation in (a) and find out the answer. (3pt)