國立中山大學	NATIONAL SUN YAT-SEN UNIVERSITY
線性代數（一）	**MATH 103 / GEAI 1215: Linear Algebra I**
第一次期中考	October 8, 2018 **Midterm 1**

姓名 Name : ____________________

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	6 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**30 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

1. [1pt] Suppose $S = \{\vec{\boldsymbol{p_1}}, \vec{\boldsymbol{p_2}}, \ldots, \vec{\boldsymbol{p_n}}\}$ is a set of n vectors over $\mathbb{R}$. Write down the definition of that "$\vec{\boldsymbol{v}}$ is a *linear combination* of vectors in S."

2. [1pt] Suppose $\vec{\boldsymbol{a}} = (a_1, \ldots, a_n)$ and $\vec{\boldsymbol{b}} = (b_1, \ldots, b_n)$ are two vectors in $\mathbb{R}^n$. Write down the definition of the *inner product* of $\vec{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}}$.

3. [2pt] Give a linear system of **three equations** in **reduced echelon form** with **three free variables**, and indicate the free variables. [The answer is not unique. You only need to find one.]

4. [2pt] Suppose $\boldsymbol{A}$ is a 5×5 nonsingular matrix. What is the minimum number of nonzero entries on $\boldsymbol{A}$? [Justify your answer with an example of such $\boldsymbol{A}$ and explain why the number of nonzero entries cannot be fewer.]

5. Let
$$\vec{\boldsymbol{u}} = (1, 1, 0, 0,) \text{ and}$$
$$\vec{\boldsymbol{v}} = \left(\sqrt{3}, \sqrt{3}, 1, 1\right).$$

(a) [1pt] Find the length $|\vec{\boldsymbol{u}}|$.

(b) [1pt] Find the length $|\vec{\boldsymbol{v}}|$.

(c) [2pt] Find the angle between $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$.

(d) [2pt] Find a vector $\vec{\boldsymbol{p}}$ that is orthogonal to both of $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$. [The answer is not unique. You only need to find one.]

6. [6pt] Find the general solution of the following linear system.

$$\begin{cases} w + 3x + \ \ y - 2z = -1 \\ 2w + 6x + 3y - 5z = -7 \\ 3w + 9x + 3y - 6z = -3 \end{cases}$$

That is, find $\vec{\boldsymbol{p}}$ and $\overrightarrow{\boldsymbol{\beta_1}}, \dots, \overrightarrow{\boldsymbol{\beta_k}}$ such that

$$\{\vec{\boldsymbol{p}} + c_1\overrightarrow{\boldsymbol{\beta_1}} + \cdots + c_k\overrightarrow{\boldsymbol{\beta_k}} : c_1, \dots, c_k \in \mathbb{R}\}$$

is the set of all solutions.

7. [6pt] Let

$$\boldsymbol{A} = \begin{bmatrix} 1 & 5 & 4 & 3 \\ 2 & 10 & 8 & 7 \\ -1 & -5 & -4 & -3 \end{bmatrix} \text{ and } \boldsymbol{R} = \begin{bmatrix} 1 & 5 & 4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

It is known that $\boldsymbol{R}$ can be obtained from $\boldsymbol{A}$ by performing some row operations. Find a matrix $\boldsymbol{C}$ such that $\boldsymbol{CA} = \boldsymbol{R}$. [The answer is not unique. You only need to find one.]

8. [6pt] Find a matrix $\boldsymbol{A}$ whose reduced echelon form is

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and every entry of $\boldsymbol{A}$ is nonzero. [The answer is not unique. You only need to find one.]

9. [extra 2pt] There are three types of 2×2 elementary matrices:

(1) $\rho_i \leftrightarrow \rho_j$: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(2) $k\rho_i$: $\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$

(3) $k\rho_i + \rho_j$: $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$

Find four matrices $\boldsymbol{E_1}, \boldsymbol{E_2}, \boldsymbol{E_3}, \boldsymbol{E_4}$ of type (2) or type (3) such that

$$\boldsymbol{E_4 E_3 E_2 E_1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

[This means the first operation (swapping) can be done by only using the second and the third operations.]

[END]

Page	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	2	
Total	30 (+2)	