Math555 Final

8 questions, 40 total points
Note: Use other papers to answer the problems. Remember to write down your **name** and your **student ID #**.

1. [5pt] Let N and X be two sets with $|N| = 2$ and $|X| = 3$. Fill in the following table by the number of functions $f : N \to X$ with the given conditions.

N	X	Any f	injective f	surjective f
dist	dist	(i)	(ii)	(iii)
indist	dist	(iv)	(v)	(vi)
dist	indist	(vii)	(viii)	(ix)
indist	indist	(x)	(xi)	(xii)

2. [5pt] Solve the recurrence relation below.

$$\begin{cases} a_n + 0a_{n-1} - 3a_{n-2} - 2a_{n-3} = 0. \\ a_0 = 4, a_1 = 3, a_2 = 17. \end{cases}$$

3. [5pt] Find the first ten terms of the reciprocal of $f(x) = 1 - x + x^2$.

4. [5pt] Let $f(x) = (1+x)^{-1}$. Find a_k such that $f''(x) = \sum_{k \geqslant 0} a_k x^k$. Here $f''(x)$ is the second (formal) derivative of $f(x)$.

5. [5pt] Compute

$$A = \sum_{n \geqslant 0} \frac{n^2}{n!}, B = \sum_{n \geqslant 0} \frac{n}{n!}, \text{ and } C = \sum_{n \geqslant 0} \frac{1}{n!}.$$

Then find the value of

$$\sum_{n \geqslant 0} \frac{2n^2 + 4n - 3}{n!}.$$

6. [5pt] Draw the Hasse diagram for the poset D_{60}.

7. [5pt] Prove that the poset D_{30} and the poset D_{105} are isomorphic. That is, find a bijection between the factors of 30 and the factors of 105 that preserve the relation.

8. [5pt] Consider the poset D_8. Find the matrix forms of the zeta function and the Möbius function on D_8, using $\{1, 2, 4, 8\}$ as the index of the matrix.