# Math555 Homework 10

**Note:** To submit the $k$-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Let $f(x) = (1+x)^{-1}$. There are two ways to compute the formal power series of $f'(x)$. Firstly, compute $f'(x)$ as a function and then expand

$$f'(x) = b_0 + b_1 x + b_2 x^2 + \cdots .$$

Secondly, write

$$f(x) = a_0 + a_1 x + x_2 x^2 + \cdots$$

and then compute the formal derivative term-by-term

$$f'(x) = c_0 + c_1 x + c_2 x^2 + \cdots .$$

Show that $b_k = c_k$ for any $k \geqslant 0$.

**Solution.** First, treating $f(x)$ as a function and get $f'(x) = -(1+x)^{-2}$. By the binomial theorem,

$$b_k = -\binom{-2}{k}.$$

In contrast,

$$f(x) = 1 - x + x^2 - x^3 + \cdots = \sum_{k \geqslant 0} \binom{-1}{k} x^k.$$

Apply the formal derivative and get

$$f'(x) = -1 + 2x - 3x^2 + \cdots = \sum_{k \geqslant 1} k \binom{-1}{k} x^{k-1} = \sum_{k \geqslant 0} (k+1) \binom{-1}{k+1} x^k.$$

Therefore,

$$\begin{aligned} c_k &= (k+1)\binom{-1}{k+1} \\ &= (k+1)\frac{(-1)(-2)\cdots(-1-k)}{(k+1)!} \\ &= -\frac{(-2)\cdots(-1-k)}{k!} = -\binom{-2}{k}. \end{aligned}$$

2. Use Sage to write a function `perm_to_inv(per)` to compute the inversion table of the permutation `perm`. Also write a function `inv_to_perm(t)` to compute the permutation for the inversion table `t`. See the file `SageProject5_blank.sagews` in your CoCalc folder.

   **Solution.** The sample solutions are posted on the course website.