# Math555 Homework 11

**Note:** To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Compute
$$A = \sum_{n\geqslant 0} \frac{n^2}{n!}, B = \sum_{n\geqslant 0} \frac{n}{n!}, \text{ and } C = \sum_{n\geqslant 0} \frac{1}{n!}.$$
Then find the value of
$$\sum_{n\geqslant 0} \frac{n^2+3n-2}{n!}.$$

**Solution.** Let $f_0(x) = e^x = \sum_{n\geqslant 0} \frac{x^n}{n!}$. Then $C = f_0(1) = e$. Next, compute
$$f_1(x) = (xD)f_0(x) = xe^x = \sum_{n\geqslant 0} \frac{nx^n}{n!}.$$
Therefore, $B = f_1(1) = e$. Again, compute
$$f_2(x) = (xD)f_1(x) = e^x + xe^x = \sum_{n\geqslant 0} \frac{n^2x^n}{n!}.$$
Thus, $A = f_2(1) = 2e$.
Finally, the desired value is
$$A + 3B - 2C = 3e.$$

2. Use Sage to calculate the formula for $\sum_{k=1}^{N} k^3$. If possible, write a function to compute the formula for $\sum_{k=1}^{N} k^p$. See the file `SageProject6_blank.sagews` in your CoCalc folder.

   **Solution.** The sample solutions are posted on the course website.