Math555 Homework 13 [Optional]

Note: To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Prove that the poset D_{12} and the poset D_{175} are isomorphic. That is, find a bijection between the factors of 12 and the factors of 175 that preserve the relation.

   Solution. Note that
   $$12 = 2^2 \cdot 3 \text{ and } 175 = 5^2 \cdot 3.$$

   The factors of 12 are
   $$S = \{2^a \cdot 3^b : 0 \leqslant a \leqslant 2, 0 \leqslant b \leqslant 1\}$$

   and the factors of 175 are
   $$T = \{5^a \cdot 7^b : 0 \leqslant a \leqslant 2, 0 \leqslant b \leqslant 1\}.$$

   Define the map $f : S \to T$ by $f(2^a \cdot 3^b) = 5^a \cdot 7^b$. One may check that f is an isomorphism between D_{12} and D_{175}.

2. Use Sage to write a function that constructs the comparability graph of the poset D_n. Then compute the height and the width of D_n for $n = 1, \ldots, 10$. See the file `SageProject8_blank.sagews` in your CoCalc folder.

   Solution. The sample solutions are posted on the course website.