Math555 Homework 14 [Optional]

Note: To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Consider the poset D_8. Find the matrix forms of the zeta function and the Möbius function on D_8, using $\{1, 2, 4, 8\}$ as the index of the matrix.

   Solution. For the zeta function,

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

   For the Möbius function,

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

2. Use Sage to write two functions `zeta_func(n)` and `moebius_func(n)`. Given a fixed n, `zeta_func(n)` should return the matrix form of the zeta function on D_n, and `moebius_func(n)` should return matrix form of the Möbius function. Note that D_n consist of all factors of n, and they are the indices of the rows/columns. As long as the row indices and column indices are following the order of the natural numbers, the output matrix will be upper-triangular. See the file `SageProject9_blank.sagews` in your CoCalc folder.

Solution. The sample solutions are posted on the course website.