Math555 Homework 2

Note: You may turn in your homework through paper work (first three weeks only) or through CoCalc. To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Show that
$$\left(\frac{n}{k}\right)^k \leqslant \binom{n}{k} \leqslant \frac{n^k}{k!} \leqslant \left(\frac{en}{k}\right)^k$$
for any integers n and k with $1 \leqslant k \leqslant n$.

2. Show that

$$n! \leqslant \frac{e\sqrt{n} \cdot n^n}{e^n}$$

for any integer $n \geqslant 1$, where e is Euler's number.

Hint: First explain that $\ln(n-1)! + \frac{1}{2}\ln n \leqslant \int_1^n \ln x \, dx$. Then remember the fact that $n! = n \cdot (n-1)!$.