Math555 Homework 3

Note: You may turn in your homework through paper work (first three weeks only) or through CoCalc. To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Suppose $\pi = b_1 b_2 \cdots b_n$. Recall that the inversion table of π is $a_1 a_2 \cdots a_n$ such that $0 \leqslant a_i \leqslant n - i$ for all i, where

$$a_{b_i} = \left|\{j < i : b_j > b_i\}\right|.$$

A left-to-right maximum of π is a digit b_j such that $b_j \geqslant b_i$ for all $i \leqslant j$. Finish the following table.

Solution. The table below lists all the 24 permutations in Σ_4 and their inversion tables.

permutations in Σ_4	inversion table	# of left-to-right maxima
1234		4
1243		
1324		
1342		
1423		
1432		
2134		
2143		
2314		
2341		
2413		
2431		
	1100	2
	1200	
	2100	
	3100	
	2200	
	3200	
	1110	
	1210	
	2110	
	3110	
	2210	
	3210	

2. Given that $s(n,k) = (-1)^{n-k}c(n,k)$ and

$$\sum_{k=0}^{n} c(n,k)x^k = (x+n+1)_n,$$

show that

$$\sum_{k=0}^{n} s(n,k)x^k = (x)_n.$$