Math555 Homework 7

Note: To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Suppose x_n is an integer for every $n \geqslant 1$ and

$$y_n = \sum_{d|n} x_d.$$

If $y_n = n^2$ for every $n \geqslant 1$. Use Möbius inversion to find x_{36} and x_{1000}. You can either do it by hand or by computer. (Send me your code in the latter case.)

Solution. By Möbius inversion,

$$\begin{aligned} x_{36} &= y_1\mu(36) + y_2\mu(18) + y_3\mu(12) + y_4\mu(9) + y_6\mu(6) \\ &\quad + y_9\mu(4) + y_{12}\mu(3) + y_{18}\mu(2) + y_{36}\mu(1) \\ &= 0 + 0 + 0 + 0 + 36 + 0 - 144 - 324 + 1296 = 864. \end{aligned}$$

Similarly, you can do $x_{1000} = 720000$.

For using a computer, here is my code in Sage as an example.

```
y(n)=n^2;
k=1000;
### below computes x_k
sum([y(d)*moebius(k/d) for d in range(1,k+1) if k % d == 0])
```

2. Use Sage to write a function that takes two inputs `k` and `B` and returns the number of ways to put `k` rooks on the board `B` in non-attacking positions. See the file `SageProject2_blank.sagews` in your CoCalc folder.

   Solution. The sample solutions are posted on the course website.