Math555 Homework 8

Note: To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Solve the recurrence relation below.

$$\begin{cases} a_n - 3a_{n-1} + 0a_{n-2} + 4a_{n-3} = 0. \\ a_0 = 3, a_1 = 2, a_2 = 14. \end{cases}$$

Solution. The characteristic polynomial is

$$p(x) = x^3 - 3x^2 + 4 = (x+1)(x-2)^2$$

with the roots $-1, 2, 2$. Thus, the formula for a_n is

$$a_n = A(-1)^n + B \cdot 2^n + C \cdot n \cdot 2^n.$$

Substituting this equality with $n = 0, 1, 2$, we get the following equations.

$$\begin{cases} A + B = 3 \\ (-1)A + 2B + 2C = 2 \\ A + 4B + 8C = 14 \end{cases}$$

It follows that $A = 2$, $B = 1$, and $C = 1$, so

$$a_n = 2(-1)^n + 2^n + n \cdot 2^n.$$

2. Use Sage to write two functions such that both compute a_n in the previous question. One uses the recurrence relation, while the other uses the formula you found. Compute the values of a_n for $n = 1, \ldots, 10$ and check if your answer is correct. See the file `SageProject3_blank.sagews` in your CoCalc folder.

Solution. The sample solutions are posted on the course website.