Math555 Homework 9

Note: To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Find the first ten terms of the reciprocal of $f(x) = 1 + x + x^2$.

   Solution. Let $g(x) = b_0 + b_1x + b_2x^2 + \cdots$. Suppose $f(x)g(x) = 1$. Direct computation gives the following.

$$\begin{aligned}
1 &= 1b_0 \implies b_0 = 1 \\
0 &= 1b_1 + 1b_0 \implies b_1 = -1 \\
0 &= 1b_2 + 1b_1 + 1b_0 \implies b_2 = 0 \\
0 &= 1b_3 + 1b_2 + 1b_1 \implies b_3 = 1 \\
0 &= 1b_4 + 1b_3 + 1b_2 \implies b_4 = -1 \\
0 &= 1b_5 + 1b_4 + 1b_3 \implies b_5 = 0 \\
0 &= 1b_6 + 1b_5 + 1b_4 \implies b_6 = 1 \\
0 &= 1b_7 + 1b_6 + 1b_5 \implies b_7 = -1 \\
0 &= 1b_8 + 1b_7 + 1b_6 \implies b_8 = 0 \\
0 &= 1b_9 + 1b_8 + 1b_7 \implies b_9 = 1
\end{aligned}$$

   Thus,

$$g(x) = 1 - x + 0x^2 + x^3 - x^4 + 0x^5 + x^6 - x^7 + 0x^8 + x^9 + \cdots.$$

2. Use Sage to write a function `reciprocal(f,k)` to compute the first `k` terms of the reciprocal of `f`. (Return `False` if `f[0]==0`.) See the file `SageProject4_blank.sagews` in your CoCalc folder.

   Solution. The sample solutions are posted on the course website.