Sample Questions 0

1. Compute the value for each of

$$A = -(2+3),$$
$$B = -2+3,$$
$$C = -2-3.$$

For two numbers a and b, find a formula equivalent to $-(a+b)$ but without parentheses.

2. Compute the value for each of

$$A = -(3-5),$$
$$B = -3-5,$$
$$C = -3+5.$$

For two numbers a and b, find a formula equivalent to $-(a-b)$ but without parentheses.

3. Compute the value for

$$A = (-2)^4 \text{ and } B = -2^4.$$

[Hint: Think about PEMDAS.]

4. Suppose we know $500x + 2 = 3$. Then $250x + 1 = ?$ [Hint: Use the distributive law to write $250x + 1$ as $k(500x + 2)$ for some number k.]

5. Suppose $a < 0$ is a negative number. By definition, $|a| = -a$, which has a negative sign. Is $-a$ a positive number or a negative number? [Hint: Pick any negative number a and compute both $|a|$ and $-a$.]

6. Without using the absolute value, what is the distance between x and 3 when $x > 3$? What about $x < 3$? [Hint: Draw a real line and mark x and 3.]

7. Solve for x that satisfies

$$|x-5| = 2.$$

8. Solve for x that satisfies

$$2|x-5| - 4 = 2.$$

9. Draw the intersection of the intervals $(-2, 3]$ and $(1, 5]$.

10. What is the intersection of $(-2, 1]$ and $(3, 5]$? Note: If a set contains nothing, then we call it an empty set, denoted as $\varnothing$.

11. Solve for x that satisfies

$$-2 < -3x + 4 \leqslant 7.$$

12. Solve for x that satisfies

$$-4x + 3 \geqslant 11 \text{ or } -5x < 5.$$

13. Solve for x that satisfies

$$2|x-3| \leqslant 14.$$

14. Solve for x that satisfies

$$2|x-3| \geqslant 14.$$

Sample Questions 1

1. Let $A = (1, 2)$ and $B = (7, -6)$ be two points on the coordinate system. Find the midpoint and the distance between A and B.

2. Find the equation of the circle centred at $(2, 3)$ with radius 5.

3. Find the equation of the circle centred at $(1, 2)$ and passing through the point $(7, -6)$.

4. Find the centre and the radius of the circle
$$x^2 - 2x + y^2 + 6y = 6.$$

5. Find the equation of the line with x-intercept $(3, 0)$ and y-intercept $(0, -4)$.

6. Find the equation of the line with slope 3 and passing through the point $(1, 2)$.

7. Find the equation of the line passing through two points $(1, 2)$ and $(5, 10)$. [Hint: You may find the slope first and then use the point-slope form.]

8. Find the equation of the line passing through two points $(1, 3)$ and $(5, 3)$.

9. Find the equation of the line passing through two points $(-1, 2)$ and $(-1, 5)$.

10. First write the line $2x+3y = 5$ in slope-intercept form. Then find the slope and the y-intercept.

11. Find the equation of the line that is parallel to $2x + 3y = 5$ and passes through the point $(4, 7)$.

12. Find the equation of the line that is perpendicular to $2x + 3y = 5$ and passes through the point $(4, 7)$.

13. Find the equation of x-axis. That is, the horizontal line passing through $(0, 0)$, $(1, 0)$, $(2, 0)$, ...

14. Find the equation of y-axis. That is, the vertical line passing through $(0, 0)$, $(0, 1)$, $(0, 2)$, ...

Sample Questions 2

1. Let $f = \{(1,2),(2,3),(3,1),(3,2)\}$. Give a reason showing f is not a function.

2. Let $g = \{(0,6),(1,5),(2,4),(3,2)\}$. Find the domain and the range of g.

3. A function is not limited to relations of numbers. We may define the function `taste` that gives a relation of an ingredient to its flavor. Consider the function

$$\texttt{taste} = \{(\texttt{salt},\texttt{salty}),(\texttt{sugar},\texttt{sweet}),\ (\texttt{pepper},\texttt{spicy}),(\texttt{lemon},\texttt{sour})\}.$$

   What are the domain and the range of `taste`?

4. Sketch the function $g(x) = \frac{2}{x-3}$. And find the domain and the range of g.

5. Sketch the function $h(x) = \sqrt{x+5} + 2$. And find the domain and the range of h.

6. Sketch the function

$$g(x) = x(20 - x).$$

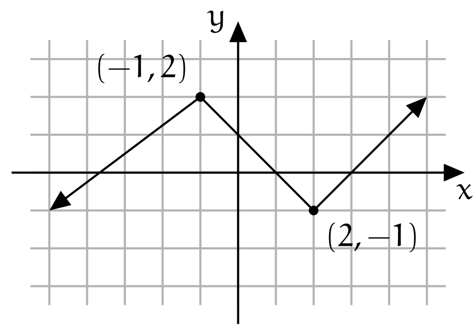
   What are the intersection(s) of the graph of g and the x-axis?

7. Sketch the function $h(x) = (x-1)^2$. Find the intersection of the graph of h and the x-axis. Find the intersection of the graph of h and the y-axis.

8. Let $f(x) = 4x+5$. Find the average rage of change of f from $x = 1$ to $x = 3$.

9. Sketch $g(x) = |x-1| + 3$. And find the domain and the range of g.

10. Let h be the function described by the graph below. Find the interval(s) where h is increasing and the interval(s) where h is decreasing.



11. Let h be the same function as the previous question. Find the value of $f(0)$? Find a value b such that $f(b) = 0$.

12. Let $f(x) = -x^2 + x + 1$. Write down the formula of $f(x-1)$.

13. Let $g(x) = x^2 + 1$. Find $g(2)$. Find $g(2+h)$ as a formula of h.

14. Let $g(x) = x^2 + 1$. The average rate of change of g from $x = 2$ to $x = 2 + h$ is

$$\frac{f(2+h) - f(2)}{(2+h) - 2}.$$

   Assuming $h \neq 0$, simplify this expression of h by expanding all the parentheses and cancelling the common factor h. (This example shows that the average rate of change is a function of h.)

Sample Questions 3

1. The equation of the unit circle centred at the origin $(0, 0)$ is $x^2 + y^2 = 1$. Move the graph to the right by 3 and to the top by 4, obtaining the unit circle centred at $(3, 4)$. Use the rule of translations and find the equation for this graph. [You may compare your answer with the standard form of the new circle.]

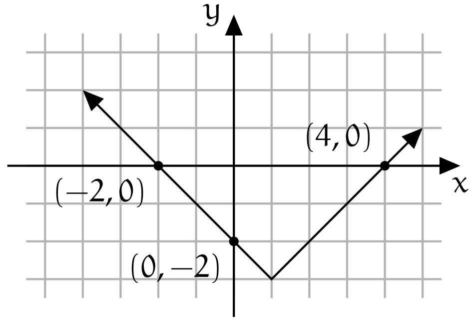
2. Let $f(x) = x^2$. Find the function whose graph is obtained from the graph of f by translating to the left by 2 and to the top by 5.

3. The equation $x^2 + y^2 = 1$ represent of the unit circle. Stretch the graph horizontally by 5 and vertically by 7. Find the equation of this new graph. [Now you know the equation of an ellipse.]

4. Let $f(x) = x^2 + 1$. Verify that f is an even function.

5. Let $f(x) = \frac{|x|}{x}$. Verify that f is an odd function.

6. Suppose the function f has the graph shown below. Solve the inequality $f(x) \leqslant 0$.



7. Solve $x(x - 20) < 0$. [Drawing the graph might help.]

8. Let $f(x) = x + 1$ and $g(x) = x^2 - x$. Let $h_1 = f + g$, $h_2 = f - g$, $h_3 = f \cdot g$, and $h_4 = \frac{f}{g}$ be four functions. Find $h_1(5)$, $h_2(5)$, $h_3(5)$, and $h_4(5)$.

9. Let $f(x) = x^2 + x$ and $g(x) = x + 1$. Find the formulas of $f \circ g(x)$ and $g \circ f(x)$. [Note that they are the compositions but not the products of f and g.]

10. Let $h(x) = \sqrt{x^2 + x} + (x^2 + x)^6$ and $f(x) = \sqrt{x} + x^6$. Find a function $g(x)$ such that $h(x) = f(g(x))$.

11. Let $h(x) = (2x + 1)^4 - \frac{1}{2x+1}$ and $g(x) = 2x + 1$. Find a function $f(x)$ such that $h(x) = f(g(x))$.

12. Let $f(x) = 3x + 4$. Find the inverse function $f^{-1}(x)$.

13. Let $f(x) = 2x + 1$ and $g(x) = \frac{x-1}{2}$. Verify that f and g are inverses of each other.

14. Let $f(x) = x^2$ be a function with domain $\mathbb{R}$ and $g(x) = \sqrt{x}$ a function with domain $[0, \infty)$. Find a value of x such that $g(f(x)) \neq x$. [Therefore, f and g are not inverses of each other. This example shows $f(g(x)) = x$ does not imply $g(f(x)) = x$.]

Sample Questions 4

1. The graph of $f(x) = 2x^2 + 12x - 5$ is a parabola. Find the vertex and the axis of symmetry of this parabola. Does this parabola open upward or open downward?

2. Find the maximum of
$$f(x) = -x^2 - 4x + 5.$$
For what value of x does this maximum occur?

3. Use the quadratic formula to find the solutions of $5x^2 + 10x + 4 = 0$.

4. Write $f(x) = x^2 + 4x + 3$ in the vertex form. Then use it to solve $f(x) = 0$.

5. The expansion of $(x + a)(x + b)$ is $x^2 + (a + b)x + ab$. Try to find two integers a and b such that $a + b = 5$ and $ab = 6$. Then use it to solve $x^2 + 5x + 6 = 0$.

6. Find two integers a and b such that
$$x^2 - 3x - 10 = (x + a)(x + b).$$
Then solve $x^2 - 3x - 10 = 0$. [This process is called *factorization*.]

7. Solve the inequality $x^2 - 6x + 8 < 0$.

8. Solve the inequality $-2x^2 + 4x + 6 \leqslant 0$.

9. Simplify i^{42} and i^{43}.

10. Let $z_1 = -17+19i$ and $z_2 = 1+5i$. Find $z_1 + z_2$, $z_1 - z_2$, and $z_1 z_2$ and simplify your answers.

11. Expand $(a + bi)(a - bi)$ and simplify it to a formula in a and b.

12. Rationalize $\frac{-17+19i}{1+5i}$.

13. Rationalize $\frac{23+2i}{4-5i}$.

14. Find the imaginary solutions of
$$x^2 + 2x + 3 = 0.$$
Sketch the graph of $f(x) = x^2 + 2x + 3$. Does the graph intersect with the x-axis? [Note this equation does not have real solutions.]

Sample Questions 5

1. Factorize $x^2 + 2x - 3$ and find all its roots.

2. Factorize $x^2 - 5x + 6$ and find all its roots.

3. Let
$$f(x) = x^3 + 3x^2 - 4x + 2.$$
First compute $f(3)$. Then use the division algorithm to find the quotient and the remainder of $f(x) \div (x - 3)$.

4. Let
$$f(x) = 2x^3 + x^2 + 5x - 2.$$
First compute $f(-2)$. Then use the division algorithm to find the quotient and the remainder of $f(x) \div (x + 2)$.

5. Let
$$f(x) = x^3 - 2x^2 - 2x + 3.$$
It is known that $f(1) = 0$. Find all (real or imaginary) roots of $f(x)$.

6. Let
$$f(x) = x^3 - 3x^2 + 4x - 2.$$
It is known that $f(1) = 0$. Find all (real or imaginary) roots of $f(x)$.

7. Use quadratic formula to find the roots of $x^2 + 4x + 2$. Then solve $x^2 + 4x + 2 < 0$.

8. Find a polynomial whose roots include $3 + 4i$.

9. Find a polynomial whose roots include 2 and $3 + 2i$.

10. Let
$$f(x) = x^3 + 4x^2 + 8x + 8.$$
According to the rational root theorem, write down all possible candidates for the rational roots of $f(x)$. Then use them to find at least one root.

11. Taking multiplicities into consideration, find all roots of
$$f(x) = x^3 + x^2 - 8x - 12.$$

12. Taking multiplicities into consideration, find all roots of
$$f(x) = x^3 - 3x^2 + 3x - 1.$$

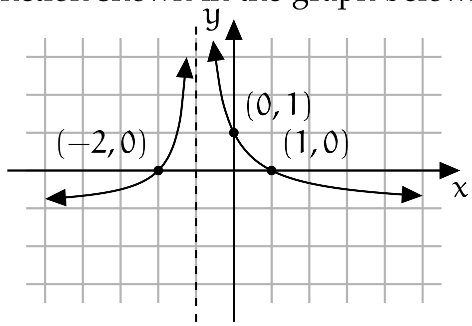
13. According Descartes's rule of sign, what are the possible number of positive roots of
$$2x^3 - x^2 + 3x - 4.$$

14. According Descartes's rule of sign, what are the possible number of negative roots of
$$2x^3 - x^2 + 3x - 4.$$

Sample Questions 6

1. Solve the equation $\sqrt{x} + 6 = x$.
2. Solve the equation $x^{\frac{4}{3}} = 16$.
3. Solve the equation $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$.
4. Solve the equation $|x^2 + 8| = 6x$.

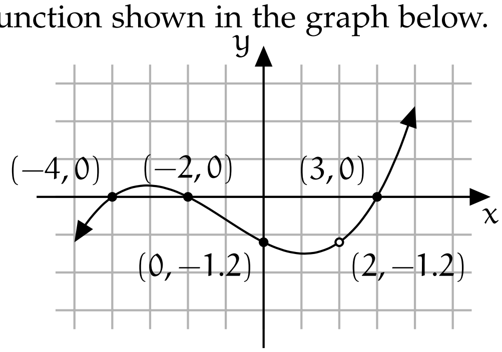
- For Questions 5 and 6, let $f(x)$ be the function shown in the graph below.



5. Find the y-intercept and the x-intercept(s) of f. Also find the values of x at which f is not defined.

6. Find the interval(s) where $f > 0$. Find the intervals(s) where $f < 0$. Find the interval(s) where f is increasing. Find the interval(s) where f is decreasing.

- For Questions 7 and 8, let $f(x)$ be the function shown in the graph below.



7. Find the y-intercept and the x-intercept(s) of f. Also find the values of x at which f is not defined.

8. Find the interval(s) where $f > 0$. Find the intervals(s) where $f < 0$. Find the interval(s) where f is increasing. Find the interval(s) where f is decreasing. [You may assume f has a peak at $x = -3$ and a valley at $x = 1$.]

- For Questions 9 ~ 11, let

$$f(x) = (x-1)(x-2)(x-3)$$
$$= x^3 - 6x^2 + 11x - 6.$$

9. Find the y-intercept and the x-intercept(s).

10. Use the one-point theorem together with the sign chart to find interval(s) where $f < 0$ and $f > 0$, respectively.

11. Use the leading term of f to determine the asymptotic behaviors of f when $x \to \infty$ and when $x \to -\infty$. Then use all the information you found in this question set to sketch the graph. (Plot more points if you like.)

- For Questions 12 ~ 14, let

$$f(x) = \frac{2(x-1)(x-2)}{3(x+1)(x-2)} = \frac{2x^2 - 6x + 4}{3x^2 - 3x - 6}.$$

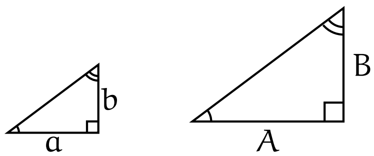
12. Find the y-intercept and the x-intercept(s). Also find the value(s) of x at which f is not defined.

13. Use the one-point theorem together with the sign chart to find interval(s) where $f < 0$ and $f > 0$, respectively.

14. Use the leading term of f to determine the asymptotic behaviors of f when $x \to \infty$ and when $x \to -\infty$. Then use all the information you found in this question set to sketch the graph. (Plot more points if you like.)

Sample Questions 7

1. The following two right triangles have the same angles. If $\frac{b}{a} = \frac{3}{4}$, find $\frac{B}{A}$. If $A = 8$, find B.



2. For each of $30°$, $60°$, $90°$, and $135°$, convert it to radian.

3. Let $\alpha = 700°$. Find the coterminal angle of α such that it is between $0°$ and $360°$. Which quadrant α is in?

4. Let $\alpha = \frac{11}{4}\pi$. Find the coterminal angle of α such that it is between 0 and 2π. Which quadrant α is in?

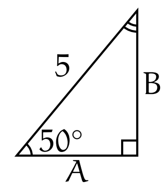
5. Write $135°$ in radian. Find the length of the arc with radius 5 and angle $135°$.

6. Find the perimeter of a circle whose radius is 5. On this circle, the length of an arc with $135°$ is the product of $\frac{135}{360}$ and the perimeter. Find the length of this arc.

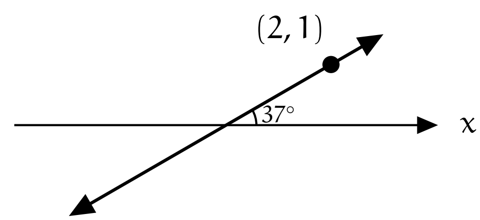
7. Find the values of $\sin(30°)$, $\sin(45°)$, $\sin(135°)$, and $\sin(150°)$.

8. Find the values of $\cos(30°)$, $\cos(45°)$, $\cos(135°)$, and $\cos(150°)$.

9. Consider the right triangle below. Find A and B. Your answers will have $\sin(50°)$ or $\cos(50°)$ involved, or you can use your calculator to find the numerical values.



10. Assume $\sin(37°) = \frac{3}{5}$ and $\cos(37°) = \frac{4}{5}$. If the angle between a line and the x-axis is $37°$, find the slope of thie line. Use the slope to find the equation of the line shown below.



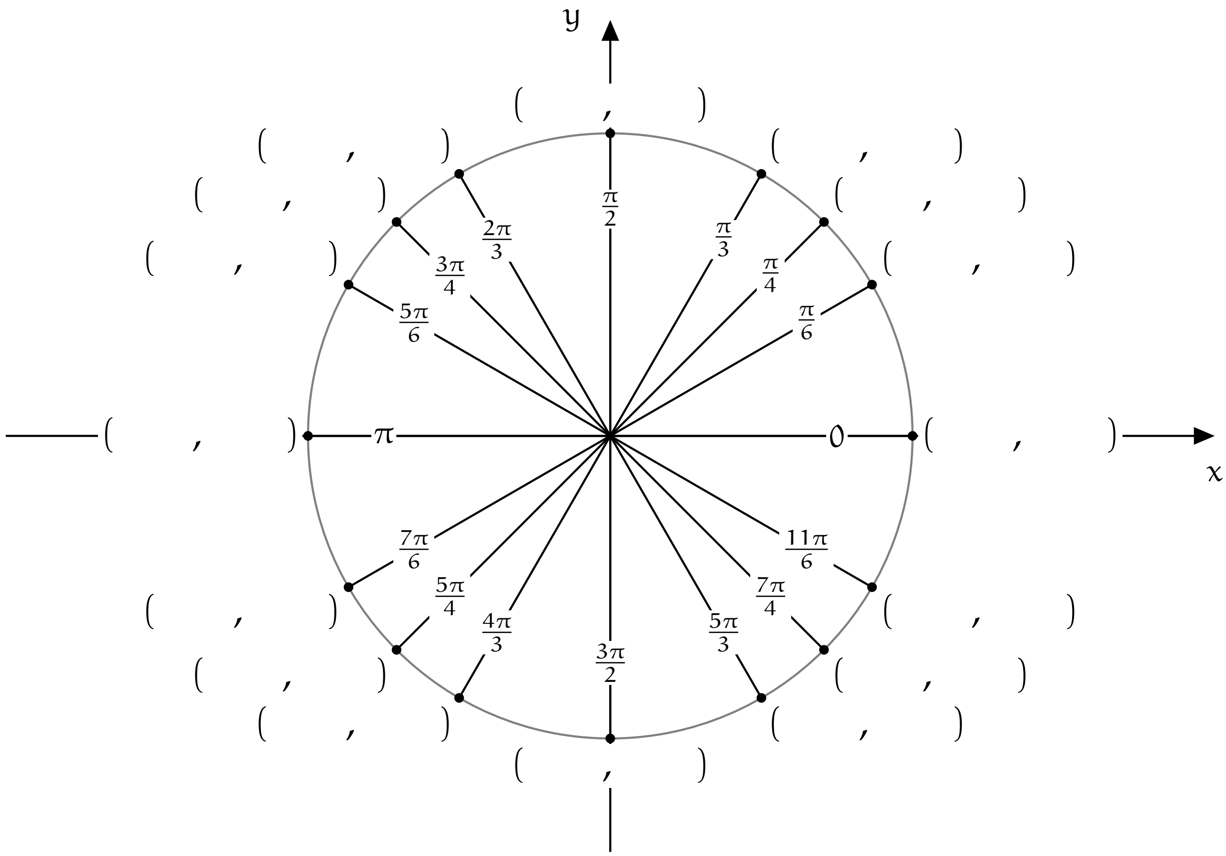
11. Suppose $\sin(\alpha) = \frac{2}{3}$ for some α that is in the second quadrant. Find $\cos(\alpha)$.

12. Find two angles α such that $\sin(\alpha) = 1$. Find two angles α such that $\sin(\alpha) = 0$. [The answers are not unique.]

13. Find two angles α such that $\cos(\alpha) = 1$. Find two angles α such that $\cos(\alpha) = 0$. [The answers are not unique.]

14. Think about the definitions of sin and cos. Draw a graph to convince yourself that $\sin(\alpha) = -\sin(-\alpha)$ and $\cos(\alpha) = \cos(-\alpha)$. Note that if α goes counterclockwisely, then $-\alpha$ is the same angle going clockwisely. Determine if sin is an even function or an odd fuction. How about cos?

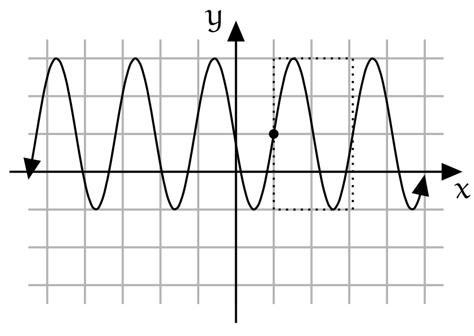
Sample Questions 8



1. The graph above is a unit circle (a circle with radius 1) and the number on each line is the angle in radian. Write down the coordinates of each of the 16 points on the unit circle.

2. Find the values of $\tan(\frac{2\pi}{3})$, $\sec(\frac{7\pi}{6})$, $\csc(\frac{4\pi}{3})$, and $\cot(\frac{11\pi}{6})$.

3. The graph of $f(x)$ is shown below. The dotted rectangle marks the building block, which has height 4 and width $\frac{2\pi}{3}$ and the reference point at $(1, 1)$. Find the period and the amplitude of $f(x)$. Using sin or cos, write down the formula of $f(x)$.



Sample Questions 9

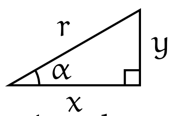
1. Find at least two angles α such that $\sin(\alpha) = \frac{\sqrt{3}}{2}$.

2. Find at least two angles α such that $\cos(\alpha) = \frac{1}{\sqrt{2}}$.

3. Find at least two angles α such that $\tan(\alpha) = \sqrt{3}$.

4. Compute $\arcsin(-\frac{1}{2})$, $\arccos(-\frac{1}{2})$, and $\arctan(\sqrt{3})$. Then compute the value of $\arccos(\cos(240^\circ))$.

- For Questions 5 $\sim$ 8, consider the graph below with α an acute angle.



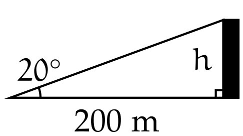
5. Suppose $x = 4$ and $y = 3$. Find the values of r, $\sin(\alpha)$, $\cos(\alpha)$, and $\tan(\alpha)$.

6. Suppose $r = 6$ and $\sin(\alpha) = \frac{1}{3}$. Find the values of x, y, $\cos(\alpha)$, and $\tan(\alpha)$.

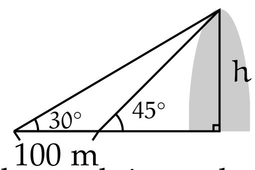
7. Suppose $x = 1$ and $\tan(\alpha) = 2$. Find the values of r, y, $\sin(\alpha)$, and $\cos(\alpha)$.

8. Suppose $y = 9$ and $\tan(\alpha) = 3$. Find the values of r, x, $\sin(\alpha)$, and $\cos(\alpha)$.

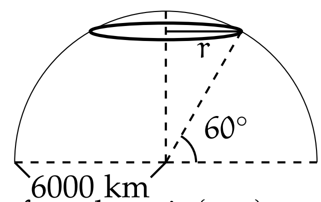
9. Use the information given in the picture below to find the height h of the building. [Round your answer to the nearest hundredth.]



10. Use the information given in the picture below to find the height h of the mountain. [Round your answer to the nearest hundredth.]



11. Suppose the earth is a sphere with radius 6000 kilometers. The points with latitude 60°N form a ring (the bolded part in the graph below). Find the radius r of this ring.



12. Use the fact that $\sin(-\alpha) = -\sin(\alpha)$ and $\cos(-\alpha) = \cos(\alpha)$ to derive the difference formula from the sum formula. That is, finish the following

$$\begin{aligned} \sin(\alpha - \beta) &= \sin(\alpha + (-\beta)) \\ &= \sin(\alpha)\cos(-\beta) + \cos(\alpha)\sin(-\beta) \\ &= \cdots \end{aligned}$$

Do the same thing for cos and tan.

13. Use the sum/difference formulas to find $\sin(75^\circ)$ and $\cos(75^\circ)$.

14. Use the formulas

$$\begin{aligned} \cos(2\alpha) &= 1 - 2\sin^2(\alpha) \\ &= 2\cos^2(\alpha) - 1 \end{aligned}$$

with $2\alpha = 45^\circ$ to find $\cos(22.5^\circ)$ and $\sin(22.5^\circ)$. Then use them to find $\cos(157.5^\circ)$ and $\sin(157.5^\circ)$.

Sample Questions 10

1. Write at least four elements in the set
$$\{30^\circ + k \cdot 360^\circ \mid k \in \mathbb{Z}\}.$$

2. Find all angles α with $\sin(\alpha) = -\frac{1}{2}$.

3. Find all angles α with $\cos(\alpha) = -\frac{1}{2}$.

4. Find all angles α with $\tan(\alpha) = -1$.

5. Use the calculator to solve $\tan(\alpha) = 3$ with $180^\circ \leqslant \alpha \leqslant 270^\circ$. [Round your answer to the nearest hundredth.]

6. Use the calculator to solve $3\cos(\alpha) = \sin(\alpha)$ with $0^\circ \leqslant \alpha \leqslant 90^\circ$. [Round your answer to the nearest hundredth.]

7. Find all angles α with
$$\frac{1}{\sqrt{2}}\sin(\alpha) + \frac{1}{\sqrt{2}}\cos(\alpha) = 1.$$

8. Find all angles α with
$$\sin^2(\alpha) + 2\sin(\alpha) + 1 = 0.$$

9. Let $f(x) = x^5$ and $g(x) = 2^x$. Use the calculator if needed to answer the following: Whether $f(10)$ or $g(10)$ is larger? Whether $f(100)$ or $g(100)$ is larger?

10. Plot $f(x) = 3^x$ and label at least four points.

11. Plot $f(x) = 2^{-x}$ and label at least four points.

12. Determine the sign of each of e^3, e^{-3}, $-e^3$, and $-e^{-3}$.

13. Find $8^{\frac{2}{3}}$ and $9^{\frac{3}{2}}$.

14. A bank offers a certificate deposit (CD) with 1.2% APY, and the interest is compounded monthly. Suppose you purchased \$1000 of this CD for 5 years. What is your ending balance?

Sample Questions 11

1. Find the values of $16^{\frac{1}{2}}$, $16^{-\frac{1}{2}}$, $16^{\frac{3}{4}}$, and 16^{0}.

2. Find the values of $\log_{16}(4)$, $\log_{16}(\frac{1}{4})$, $\log_{16}(8)$, and $\log_{16}(1)$.

3. Find the values of $\log_2(\sqrt{2})$, $\ln(e^{\pi})$, $2^{\log_2(5)}$, and $e^{\ln(3)}$.

4. Let $f(x) = 2^x$ and $g(x) = e^{x \cdot \ln 2}$. Find the values of $f(0)$, $g(0)$, $f(1)$, and $g(1)$. [They are all integers and can be computed by hand.]

5. Provided that $e = 2.718\cdots$, is $\ln(3)$ greater than 1 or less than 1?

6. Draw the graph of $f(x) = \log_2(x)$ and label at least four points.

7. Draw the graph of $f(x) = \log_2(x+1)$ and label at least four points.

8. Find the values of $\frac{\ln(2^3)}{\ln(2)}$ and $\log_2(\sqrt[3]{2}) + \log_2(\sqrt[3]{4})$. [They are all integers and can be computed by hand.]

9. Use the calculator to find $\frac{\log_{10}(5)}{\log_{10}(2)}$, $\frac{\ln(5)}{\ln(2)}$, and $\log_2(5)$.

10. Solve $\log_2(3x+1) = 4$.

11. Solve $\ln(2x+1) = \ln(3x-1)$.

12. Solve
$$\ln\left(\frac{2x+1}{3x-1}\right) = 0.$$

13. Solve $2^{3x+2} = 3^{2x+1}$.

14. Solve $\ln(x^3) - \ln(x^2) = 3$.