

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數 (一)

MATH 103 / GEAI 1215: Linear Algebra I

第一次期中考

October 14, 2019

Midterm 1

姓名 Name : \_\_\_\_\_

學號 Student ID # : \_\_\_\_\_

Lecturer: Jephian Lin 林晉宏

Contents: cover page,  
**6 pages** of questions,  
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **25 points** + 2 extra points

**Do not open this packet until instructed to do so.**

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答



6. Let

$$\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ \sqrt{5} \\ 0 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

(a) [1pt] Find the length  $|\mathbf{u}|$ .

(b) [1pt] Find the length  $|\mathbf{v}|$ .

(c) [1pt] Find the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .

(d) [2pt] Find a vector  $\mathbf{w}$  such that the angle between  $\mathbf{w}$  and  $\mathbf{v}$  is  $\frac{\pi}{4}$ . [The answer is not unique. You only need to find one.]

7. [5pt] Find the general solution of the following linear system.

$$\begin{cases} w + x - 2y & = -2 \\ 5w + 5x - 10y + z & = -15 \\ 9w + 9x - 18y + 2z & = -28 \end{cases}$$

That is, find  $\mathbf{p}$  and  $\beta_1, \dots, \beta_k$  such that

$$\{\mathbf{p} + c_1\beta_1 + \dots + c_k\beta_k : c_1, \dots, c_k \in \mathbb{R}\}$$

is the set of all solutions.

8. [5pt] Let

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & -5 & 4 \\ -1 & -4 & 6 & -1 \\ 3 & 12 & -13 & 19 \end{bmatrix} \text{ and } \mathbf{R} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

It is known that  $\mathbf{R}$  is the reduced echelon form of  $\mathbf{A}$ . Write the row  $[1 \ 4 \ 0 \ 0]$  as a linear combination of rows of  $\mathbf{A}$ .

9. [5pt] Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 9 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}.$$

Is it possible to obtain  $\mathbf{B}$  from  $\mathbf{A}$  by some row operations? [This is a yes-or-no question, but you have to justify your answer.]

10. [extra 2pt] It is known that the following row operations are correct.

$$\begin{bmatrix} 13 \\ 23 \end{bmatrix} \xrightarrow{-\rho_1 + \rho_2} \begin{bmatrix} 13 \\ 10 \end{bmatrix} \xrightarrow{-\rho_2 + \rho_1} \begin{bmatrix} 3 \\ 10 \end{bmatrix} \xrightarrow{-3\rho_1 + \rho_2} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Find two integers  $a$  and  $b$  such that  $a \cdot 13 + b \cdot 23 = 1$ .

**[END]**

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	2	
Total	25 (+2)	