國立中山大學	NATIONAL SUN YAT-SEN UNIVERSITY
線性代數（一）	**MATH 103 / GEAI 1215: Linear Algebra I**
第一次期中考	October 14, 2019 **Midterm 1**

姓名 Name : ____________________

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	6 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**25 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

1. [1pt] Write down an example of a system of **linear** equations in variables a, b, and c.

2. [1pt] Write down an example of a system of equations in variables a, b, and c that is **not a linear system**.

3. [1pt] Write down an example of a system of **three linear equations** in its **echelon form** that contains **two free variables**.

4. [1pt] Write down an example of a 4×4 **nonsingular** matrix.

5. [1pt] Write down an example of a 4×4 **singular** matrix.

6. Let

$$\mathbf{u} = \begin{bmatrix} 2 \\ 3 \\ \sqrt{5} \\ 0 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

(a) [1pt] Find the length $|\mathbf{u}|$.

(b) [1pt] Find the length $|\mathbf{v}|$.

(c) [1pt] Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

(d) [2pt] Find a vector $\mathbf{w}$ such that the angle between $\mathbf{w}$ and $\mathbf{v}$ is $\frac{\pi}{4}$. [The answer is not unique. You only need to find one.]

7. [5pt] Find the general solution of the following linear system.

$$\begin{cases} \quad w - 2x - \ \ 2y + \ \ 7z = -12 \\ -2w + 4x + \ \ 5y - 19z = 28 \\ -4w + 8x + 11y - 43z = 60 \end{cases}$$

That is, find $\mathbf{p}$ and $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_k$ such that

$$\{\mathbf{p} + c_1\boldsymbol{\beta}_1 + \cdots + c_k\boldsymbol{\beta}_k : c_1, \ldots, c_k \in \mathbb{R}\}$$

is the set of all solutions.

8. [5pt] Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 & -18 \\ -1 & -1 & -1 & 3 \\ 4 & 4 & 5 & -7 \end{bmatrix} \text{ and } \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 5 \end{bmatrix}.$$

It is known that $\mathbf{R}$ is the reduced echelon form of $\mathbf{A}$. Write the row $\begin{bmatrix} 1 & 0 & 0 & -3 \end{bmatrix}$ as a linear combination of rows of $\mathbf{A}$.

9. [5pt] Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 9 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 8 & 9 \end{bmatrix}.$$

Is it possible to obtain **B** from **A** by some row operations? [This is a yes-or-no question, but you have to justify your answer.]

10. [extra 2pt] It is known that the following row operations are correct.

$$\begin{bmatrix} 13 \\ 23 \end{bmatrix} \xrightarrow{-\rho_1+\rho_2} \begin{bmatrix} 13 \\ 10 \end{bmatrix} \xrightarrow{-\rho_2+\rho_1} \begin{bmatrix} 3 \\ 10 \end{bmatrix} \xrightarrow{-3\rho_1+\rho_2} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Find two integers a and b such that $a \cdot 13 + b \cdot 23 = 1$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	2	
Total	25 (+2)	