國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（一） MATH 103 / GEAI 1215: Linear Algebra I

第二次期中考 November 25, 2019 **Midterm 2**

姓名 Name : ____________________

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	7 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**30 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

– Enter your **Name** and **Student ID #** before you start.

– Using the calculator is not allowed (and not necessary) for this exam.

– Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

– Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

– 可用中文或英文作答

1. [1pt] Let $S \subseteq \mathbb{R}^3$ be a set of vectors. Give an example of S such that $\text{span}(S) = \mathbb{R}^3$ and S is not linearly independent.

2. [1pt] Let $S \subseteq \mathbb{R}^3$ be a set of vectors. Give an example of S such that S is linearly independent and $\text{span}(S) \neq \mathbb{R}^3$.

3. [1pt] Let $S \subseteq \mathbb{R}^3$ be a set of vectors. Give an example of S such that S is linearly independent and $\text{span}(S) = \mathbb{R}^3$.

4. [1pt] Let $V \subseteq \mathbb{R}^3$ be a set of vectors. Give an example of V such that V is a not subspace of $\mathbb{R}^3$.

5. [1pt] Let $V \subseteq \mathbb{R}^3$ be a set of vectors. Give an example of V such that V is a subspace of $\mathbb{R}^3$.

6. [5pt] Find the inverse of the matrix

$$\boldsymbol{A} = \begin{bmatrix} 1 & 5 & 0 & -1 \\ -3 & -14 & 3 & 1 \\ 10 & 48 & -5 & -4 \\ 37 & 178 & -18 & -16 \end{bmatrix}.$$

7. [5pt] Let

$$V = \operatorname{span}\left(\left\{ \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 9 \\ 15 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 16 \\ 25 \end{bmatrix}, \begin{bmatrix} -23 \\ -73 \\ -115 \end{bmatrix} \right\}\right).$$

Find **a basis** and **the dimension** of V.

8. Let

$$\boldsymbol{A} = \begin{bmatrix} 1 & 3 & 1 & 5 & -23 \\ 3 & 9 & 3 & 16 & -73 \\ 5 & 15 & 5 & 25 & -115 \end{bmatrix}.$$

(a) [2pt] Find **a basis** and **the dimension** of the row space of $\boldsymbol{A}$.

(b) [3pt] Find **a basis** and **the dimension** of the null space of $\boldsymbol{A}$.

9. [5pt] Let

$$\mathbf{A} = \begin{bmatrix} ? & ? & ? & ? & a_5 \\ ? & a_2 & ? & ? & 0 \\ ? & 0 & a_3 & ? & 0 \\ a_1 & 0 & 0 & ? & 0 \\ 0 & 0 & 0 & a_4 & 0 \end{bmatrix}$$

be a 5×5 real matrix such that $a_1, \ldots, a_5$ are nonzero and each question mark represents an unknown value. Show that the columns of $\mathbf{A}$ form a linearly independent set.

10. [5pt] Let $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ be a basis of some vector space V. Let $\mathbf{y}_1 = \mathbf{x}_1 - \mathbf{x}_2$, $\mathbf{y}_2 = \mathbf{x}_1 + \mathbf{x}_2$, and $\mathbf{y}_3 = \mathbf{x}_3$. Show that $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$ is also a basis of V.

11. [extra 2pt] Let

$$f_1 = \frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)},$$
$$f_2 = \frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)},$$
$$f_3 = \frac{(x-1)(x-2)(x-4)}{(3-1)(3-2)(3-4)}, \text{ and}$$
$$f_4 = \frac{(x-1)(x-2)(x-3)}{(4-1)(4-2)(4-3)}$$

be four polynomials. Show that any polynomial f of degree at most 3 can be written as a linear combination of $f_1, \dots, f_4$. That is, for a given

$$f = a_0 + a_1 x + a_2 x^2 + a_3 x^3,$$

find coefficients $c_1, \dots, c_4 \in \mathbb{R}$ such that

$$f = c_1 f_1 + c_2 f_2 + c_3 f_3 + c_4 f_4.$$

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	2	
Total	30 (+2)	