Sample Questions 1

1. Find an echelon form for each of the linear systems. How many solutions does the system have? [unique, none, or many]

   (a) $\begin{cases} x + y = 5 \\ x - 4y = 0 \end{cases}$

   (b) $\begin{cases} -x - y = 1 \\ -3x - 3y = 2 \end{cases}$

   (c) $\begin{cases} \phantom{2x - {}} 4y + z = 20 \\ 2x - 2y + z = 0 \\ 2x + 2y + 2z = 20 \\ 2x - 6y \phantom{{}+2z} = -20 \end{cases}$

2. Find the value k such that the following linear system has many solutions.

$$\begin{cases} x - y = 1 \\ 3x - 3y = k \end{cases}$$

3. Find the value k such that the following linear system has many solutions.

$$\begin{cases} x + 2y + 3z = 10 \\ 2x - 2y + z = 5 \\ x + 8y + 8z = k \end{cases}$$

4. Find the condition(s) for the b's so that the following linear system has at least a solution.

$$\begin{cases} x - 3y = b_1 \\ 3x + y = b_2 \\ x + 7y = b_3 \\ 2x + 4y = b_4 \end{cases}$$

5. Find the coefficients a, b, and c so that the graph of $f(x) = ax^2 + bx + c$ passes through the points $(1, 2)$, $(-1, 6)$ and $(2, 3)$.

6. Suppose $\begin{cases} x + y = 1 \\ 4x - y = 6 \end{cases}$. Can you derive $11x + y = 27$? That is, find a and b the following equality holds.

$$\begin{array}{rrcrcrl} & x & + & y & = & 5 & [\times a] \\ +) & 4x & - & y & = & 6 & [\times b] \\ \hline & 11x & + & y & = & 27 & \end{array}$$

7. Four positive integers are given. Select any three of the integers, find their arithmetic average, and add this result to the fourth integer. Thus the numbers 29, 23, 21, and 17 are obtained. What are the four original integers?