Sample Questions 3

1. Show that
$$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$$
for any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^n$.

2. Show that the Cauchy–Schwartz inequality implies the triangle inequality.

3. Let
$$\mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \text{ and } X = \left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \end{bmatrix} \right\}.$$
Can $\mathbf{v}$ be written as a linear combination of vectors in X?

4. Let
$$\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \text{ and } X = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 2 \end{bmatrix} \right\}.$$
Can $\mathbf{v}$ be written as a linear combination of vectors in X?

5. Find the reduced echelon form of the augmented matrix and find the general solution of the following linear system.
$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 5 \end{bmatrix}$$

6. Find the reduced echelon form of the augmented matrix and find the general solution of the following linear system.
$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

7. Find the reduced echelon form of the augmented matrix and find the general solution of the following linear system.
$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & -1 & 1 & 1 \\ 3 & 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$