

## Sample Questions 4

1. A 0,1-matrix is a matrix whose entries are either 0 or 1. Among all  $2 \times 3$  matrices, how many 0,1-matrices are in reduced echelon form?
2. Find a  $3 \times 4$  matrix whose leading variables falls on the first and the third columns and whose entries are all nonzero. (The answer is not unique. Remember that the leading variables are observed by the echelon form but not the matrix itself.)
3. Define a relation  $\sim$  on integers by  $a \sim b$  if  $a - b$  is a multiple of 3. Prove that  $\sim$  is an equivalence relation.
4. Let  $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 11 & 14 \\ -8 & -10 \end{bmatrix}$ , and  $C = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$ . You may verify that  $CA = B$ . Write  $\begin{bmatrix} 11 & 14 \end{bmatrix}$  as a linear combination of rows of  $A$ .
5. The matrix  $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$  has the reduced echelon form  $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ . Write the row vector  $\begin{bmatrix} 0 & 1 \end{bmatrix}$  as a linear combination of rows of  $A$ .
6. The matrix  $A = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 11 & 2 \\ 8 & 17 & 2 \end{bmatrix}$  has the reduced echelon form  $R = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ . Write the row vector  $\begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$  as a linear combination of rows of  $A$ .
7. Suppose  $A$ ,  $B$ , and  $C$  are three matrices such that  $AC = B$ . Show that each column of  $B$  is a linear combination of columns of  $A$ .